197

HAND WRITTEN NOTES
MECHANICAL ENGINEERING

MACHINE DESIGN(M.D)

Design of Machine Elements

(or) MACHINE DESIGN (M.D)

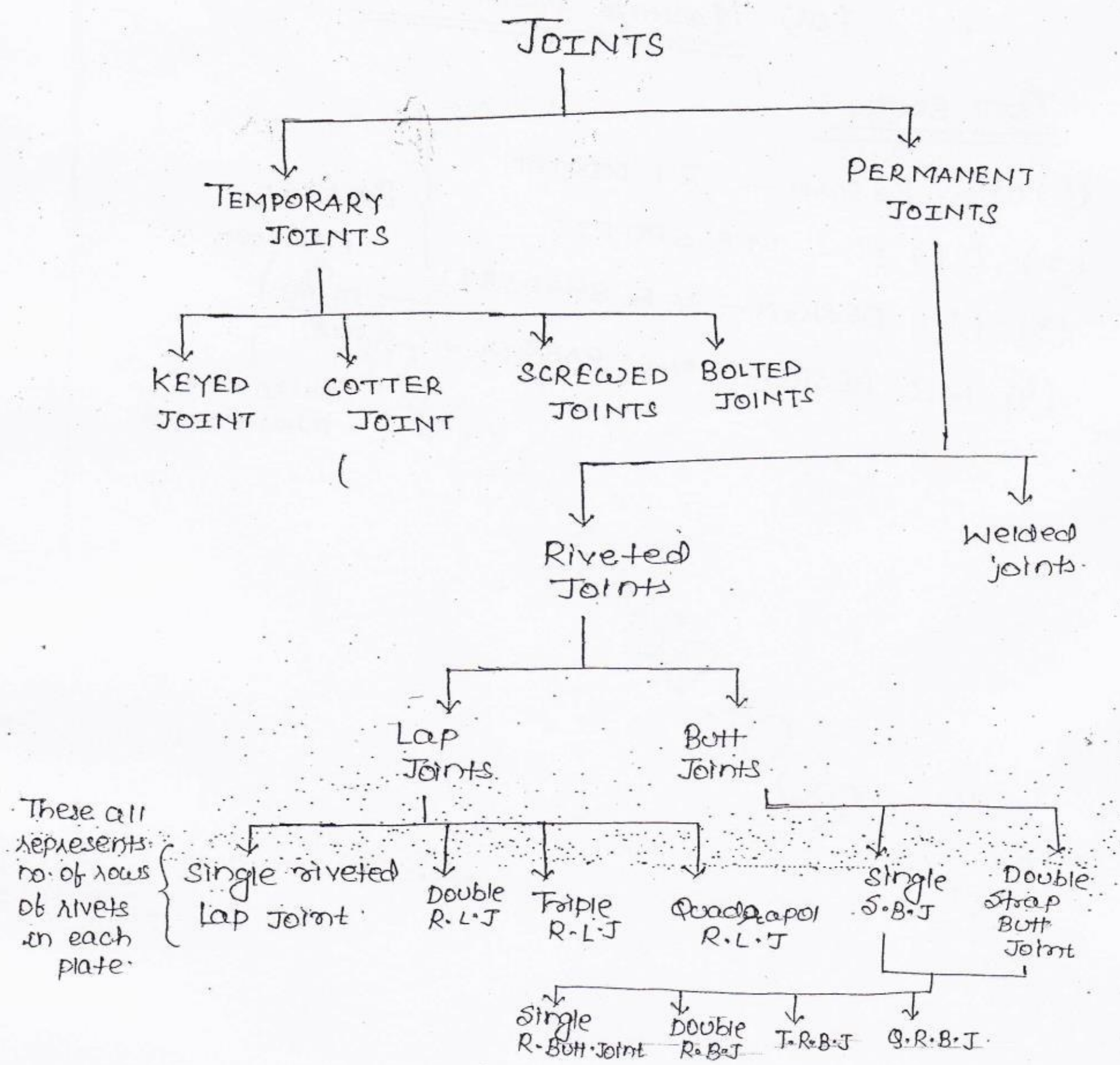
TEXT BOOKS

(1) M/C DESIGN — R.L NORTON
(2) D.M.E — M.F SPOTTS
PEARSON EDITION

(3) M/C DESIGN — V.B BHADARI → T.M.H

(4) M/C DESIGN — N.C. PANDYA & C.S SHAH → CHAROTAR PUBLICATION.

RIVETED JOINT

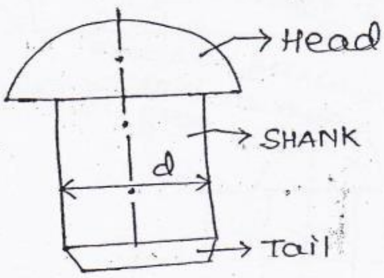


- key is used to join the two shaft and for power transmission (temporary fastener) from shaft to puly.
- Cotter is used to transmit axial compressive or tensile load and also used to join two rods (coaxial Drive rod).

Application of Rivet Joints

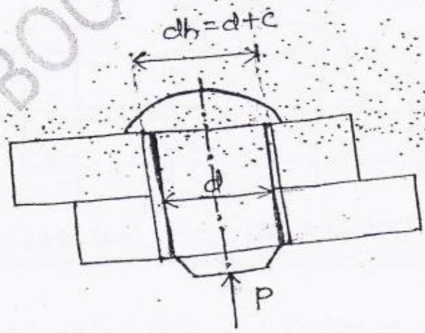
- Machine body Building applications
- Structural applications, sheet metal joints
  ↓
  Bridges, roof construction.
- Pressure vessel application.

RIVETS SPECIFICATION



(1) Rivet (or) Shank dia
(2) Type of head

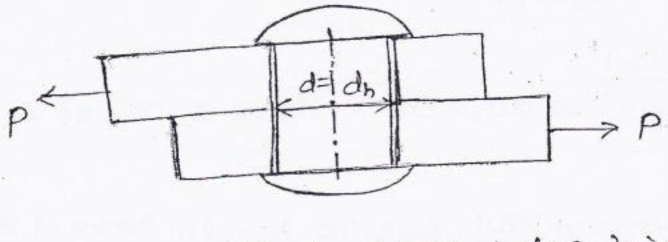
- Rivet head [refer from M.Drawing Book for P.SU & ES].
- Rivet hole is slightly larger than the Rivet dia.



If both dia of hole is given then,

In the design of rivet → d

In the design of plate → d_h.



Single riveted lap joint

In the design of the rivet, rivet or shank dia should be taken into consideration.

In the design of plates, hole dia. (d_h)

$$\boxed{d_h = d + C}$$

Should be taken into consideration. where,

C → clearance b/w the dia. of hole and dia. of rivet.

Terminology

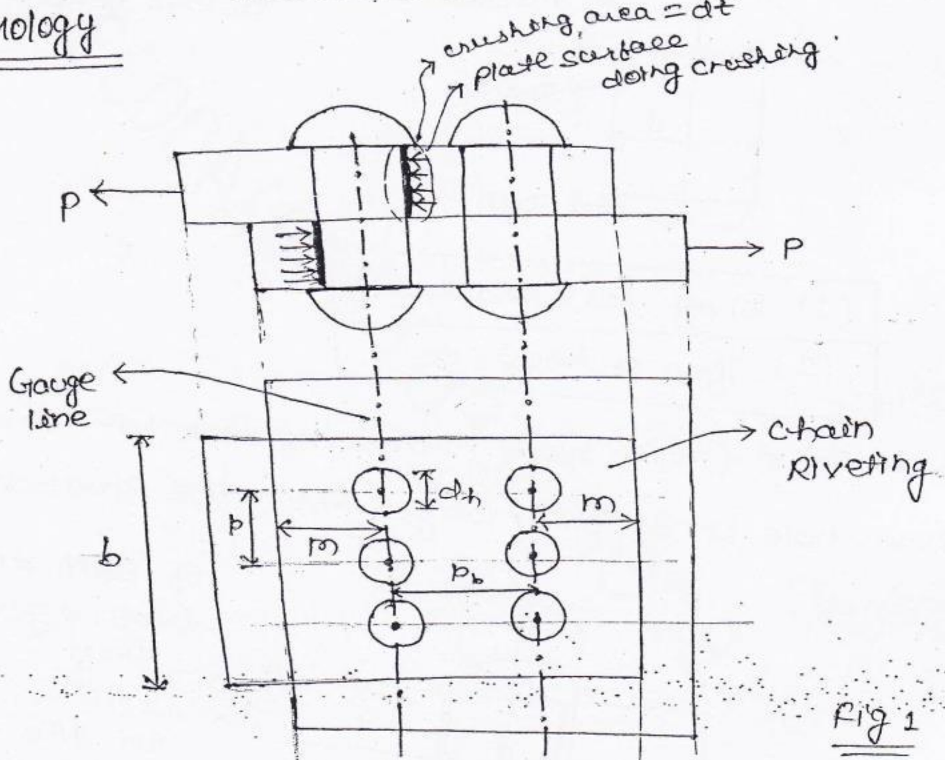


Fig 1

DOUBLE RIVET LAP JOINT

Pitch (p)

Distance measured along a gauge line b/w the centres of Adjacent rivets.

Back pitch (P_b)

Distance b/w adjacent rows of rivet or adjacent gauge line.

Gauge line

An imaginary line passing through centre of the rivet & parallel to edge of the rivet

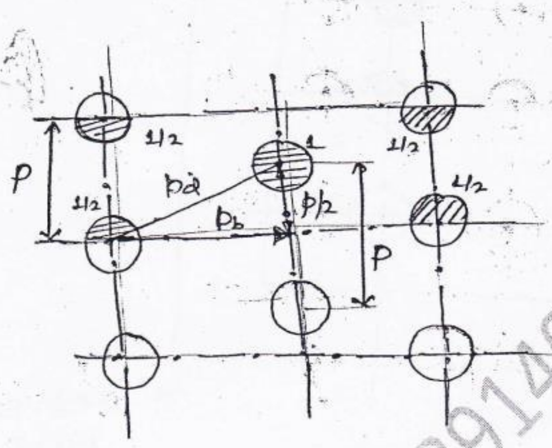
Margin (m)

Distance of gauge line to the nearest edge of the plate

Chain Rivets

If the centres of the rivets or rivets in the adjacent gauge lines are placed exactly opposite to each other it is known as chain rivets.

Zig-zag Rivets



No. of rivet per pitch length

$\frac{1}{2} \times 4 + 1$

$= 3$

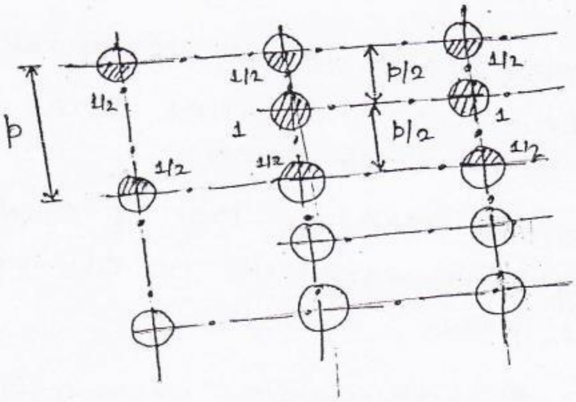
If the rivets in adjacent gauge line placed diagonally opposite to each other.

Diagonal pitch (p_d)

Distance b/w centres of adjacent rivets in the gauge line.

$$\boxed{p_d = \sqrt{p_b^2 + \left(\frac{p}{2}\right)^2}}$$

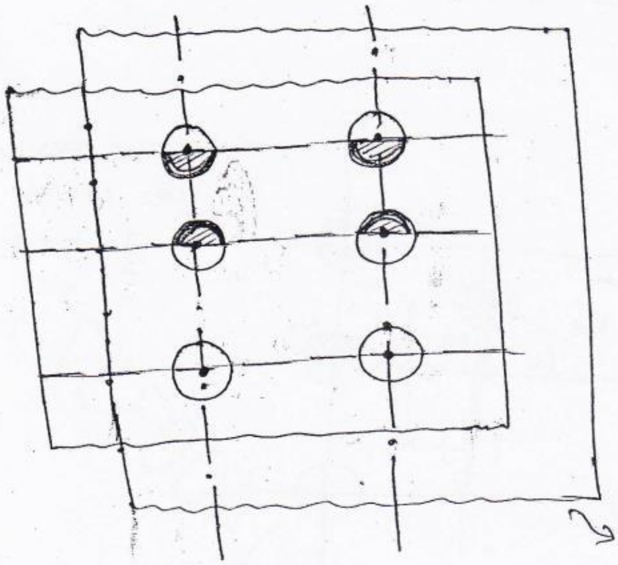
Chain riveting with unequal pitches



$n = \frac{1}{2} \times 6 + 2$

$= 5$

$$\boxed{p_1 = p_b/2}$$



No. of Rivet per pitch = $\frac{1}{2} \times 4$
$= 2$.

represent at part of plate means continuous plate (we can not determine no. of rivets)

n = No. of Rivets per pitch length.

N = Total no. of rivets in each plate.

- In case of equal pitch n is equal to no. of rows of rivets but in case of unequal pitch n is not equal to no. of rows of rivets.

(8) Quadrapol riveted joint value of n for

$$p, \; p/2, \; p/3, \; p/4.$$

$$n = 1 + 2 + 3 + 4 = 10$$

- No. of rivet per pitch length is equal to no. of rivets lying b/w two parallel lines which are drawn at a distance of pitch.
- In case of equal pitches no. of ~~pitches~~ rivet per pitch length is equal to no. of rows of rivets in each plate.

for eg:- In a Quadrapole riveted joint

$$n = 4.$$

Double riveted joint $n = 2$.

Tensile failure occur.

final width of plate → $P - 3d_R$ (Fig 1) .

for rivet crushing failure occur:

12	01	14

Efficiency of Riveted joint

(i) Dia of Rivet (d)

$$\boxed{d = 6\sqrt{t}} \Rightarrow \cancel{t \geq 8mm}$$

$\underbrace{\quad}_{\text{unwin's formula}}$

t = thickness of plates.

(ii) $\boxed{m = 1.5d}$

(iii) Strength of Rivets

P_s = shear strength of Rivets (or) shear strength of Rivet per pitch length

P_c = crushing of Rivets.
= crushing of Rivets per pitch length.

$$\tau_{ind} \leq \tau_{per}$$

$$\frac{P/N}{k\left(\frac{\pi}{4}d^2\right)} \leq \tau_{per}$$

$$\boxed{P \leq NK\frac{\pi}{4}d^2\tau_{per}}$$

$$\boxed{P_s = NK\left(\frac{\pi}{4}d^2\right)\tau_{per} \text{ (or) } nK\frac{\pi}{4}d^2\tau_{per}}$$

$$\boxed{P_c = N(dt)(\sigma_c)_{per} \text{ (or) } n(dt)(\sigma_c)_{per}}$$

where,

N = Total no. of Rivets in each plate.

n = No. of rivets per pitch length.

K = 1 ⇒ Single shear (i.e, Lap joints, Single strap Butt joint)

= 2 ⇒ Double shear (Double shear butt joint)

$$\boxed{\begin{array}{l}\text{Strength of the Rivets}\\ \text{(or) Strength of the rivets/Pitch length}\\ \quad = \text{Min. of } [P_s \text{ \& } P_c]\end{array}}$$

(4) P_t = tearing strength of the plate
(or) tearing strength of the plate/pitch length.

$$\boxed{P_t = (b - N_R d_h)(t)(\sigma_t)_{per}}$$ valid only for equal pitches.

where, N_R → No. of rivets in each row.

valid only for equal pitches

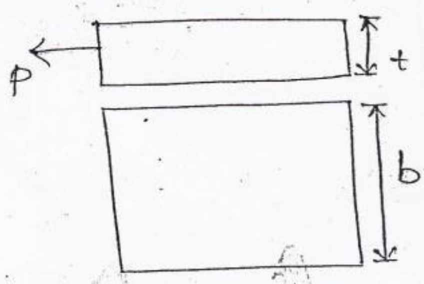
$$\boxed{P_t = (p - d_h)\, t\, (\sigma_t)_{per}}$$ (when width of the plate is unknown)

(5) Strength of the riveted joint per pitch length or strength of Riveted joint

$$= \text{Min}\,[P_c, P_s \text{ \& } P_t]$$

Output → Riveted joint
Input → Solid plate.

(6) Strength of the solid plate
(or) strength of the solid plate / Pitch length
$= P_{solid}$.



$$\frac{P}{bt} \le (\sigma_t)_{per}$$

$$\boxed{P \le (bt)(\sigma_t)_{per}}$$

$\rightarrow P_{solid}$

$$\boxed{P_{solid} = b \times t \times (\sigma_t)_{per} \quad (or) \quad p \times t \times (\sigma_t)_{per}}$$

(7) $$\boxed{\eta_{Riveted\ joint} = \frac{\text{Strength of the riveted joint}}{\text{Strength of the solid plate}} \times 100}$$

Disadvantages of riveted joint

- Always efficiency of riveted joint is not 100%.
- wt. is high due to presence of straps.
- 100% leak proof joint is not possible.

$$\eta_{tearing} = \frac{P_t}{P_{solid}} \times 100$$

$$= \frac{(b - N_R d_h) t (\sigma_t)_{per}}{b \times t \times (\sigma_t)_{per}} \times 100$$

$$= \frac{(p - d_h) t (\sigma_t)_{per}}{p \times t \times (\sigma_t)_{per}} \times 100$$

$$\boxed{\eta_{tearing} = \left(1 - \frac{d_h}{p}\right) \times 100}$$

- $$\boxed{\eta_{shearing} = \frac{P_s}{P_{solid}} \times 100}$$

$$\boxed{\eta_{crushing} = \frac{P_c}{P_{solid}} \times 100}$$

If thickness of the plate < 8mm

$$P_s = P_c$$

$$n K \frac{\pi}{4} d^2 \tau_{per} = N (dt) (\sigma_c)_{per}$$

$$\boxed{d = \frac{4t (\sigma_c)_{per}}{K \pi \tau_{per}}}$$

Assumptions made for above expressions:-
- Material is same.
- Dia is same.
- load is equally distributed.

- when load is equally distributed
when the line of action of load is passing through the centre of gravity of the group of the rivets.
- Because of this stress concentration occur.
- The following assumptions are made in the design of Riveted joint:-

(i) Line of action of load is assumed to pass through the C.G of group of Rivets i.e, applied load is equally distributed among all the rivets.

(ii) Dia of all the rivets is assumed to be the same.

(iii) Effect of stress concentration in the plates due to Rivet hole is neglected.

(iv) Bending stress developed in the rivets is neglected.

(v) Rivets completely fill the rivet hole after riveting.

(vi) Double shear area is assumed as 1.875 times the single shear area.

(vii) Frictional forces developed at the interface of the plates is neglected.

(Q) Two plates of thickness 5mm are joined by double strap double riveted butt joint. The diameter of rivet is 7mm, clearance b/w the rivet hole and the rivet is 1mm, pitch is 28mm assume permisil $(\sigma_t)_{per}$, $(\tau_s)_{per}$, $(\sigma_c)_{per}$ 120 MPa, 100MPa, 150 MPa resp. Determine the following

(i) Efficiency of Riveted joint.

(ii) Tearing efficiency.

(iii) Total no. of rivets in the joint, if load acting on the joint is 100 KN

Sol:- $t = 5mm$, $K = 2$,

Double strap Double rivetted butt joint.

$K = 2$, $n = 2$

$$P_s = nK\frac{\pi}{4}d^2 \tau_{per}$$

$$= 2 \times 2 \frac{\pi}{4}(7)^2 \times (100) = 15.39\ KN$$

(2) $P_c = ndt\,(\sigma_c)_{per}$

$$= 2 \times 7 \times 5\,(150)$$

$$= 10.5\ KN.$$

(3) $P_t = (p - d_h)t\,(\sigma_t)_{per}$ $\left(d_h = d + c = 8\ mm\right)$

$$= (28 - 8)\,5 \times 120$$

$$= 12\ KN.$$

(4) Strength of rivet joint / Pitch length

$$= \text{Min. of } [P_s, P_c, P_t]$$

$$= P_c = 10.5\ KN.$$

(5) $P_{solid/pitch\ length} = p \times t\ (\sigma_t)_{per}$

$$= 28 \times 5 \times 120$$

$$= 16.8\ KN.$$

(6) $$\eta_{Rivet\ Joint} = \frac{\text{Strength of the R.J/P.L}}{P_{solid/p.L}} \times 100$$

$$= 62.5\%$$

(7) $$\eta_{tearing} = \frac{P_t}{P_{solid}} \times 100 \ (or) \left[1 - \left(\frac{d_h}{p}\right)\right] \times 100$$

$$= 71.4\%$$

(8) N = Total no. of Rivets in each plate

$$N = \frac{\text{Total load}}{\text{Strength of each rivet}}$$

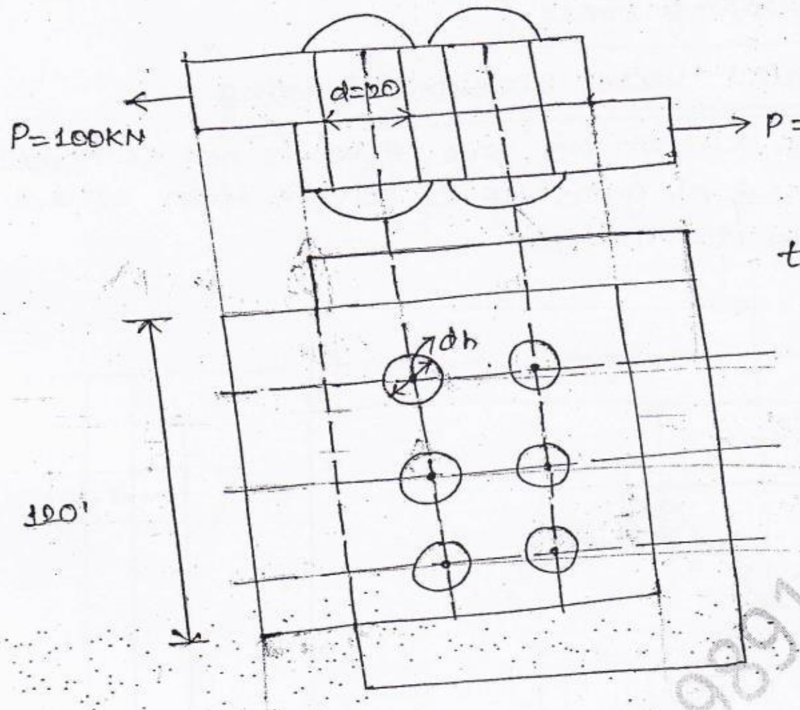
Strength of each rivet

$$= \frac{1}{n}\left[\text{Min. of } (P_s \text{ or } P_c)\right]$$

$$= \frac{1}{2}(10.5) = 5.25\ KN$$

$$N = 19.04 = 20.$$

(9) For the double riveted lap joint as shown in the figure. Determine shear stress, crushing stress developed in each rivet and tensile stress developed in the plate. Assume dia. of rivets is equal to 20mm.



t = 10mm
clearance b/w hole & rivet is 2mm.

$P_s = N K \frac{\pi}{4} d^2 \tau_{per}$

$100\times10^3 = 6\times1\times\frac{\pi}{4}(20)^2 \tau_{ind}$

$\tau_{ind} = \frac{4\times100\times10^3}{6\times1\times\pi\times(20)^2} = 53.07\ MPa.$

$P_c = Ndt\ \sigma_c$

$100\times10^3 = 6\times20\times10\times\sigma_c$

$\Rightarrow \sigma_c = \frac{100\times10^3}{6\times20\times10} = 83.3\ MPa.$

$P_t = (b - N_R d_h)\, t\, (\sigma_t)_{per}$

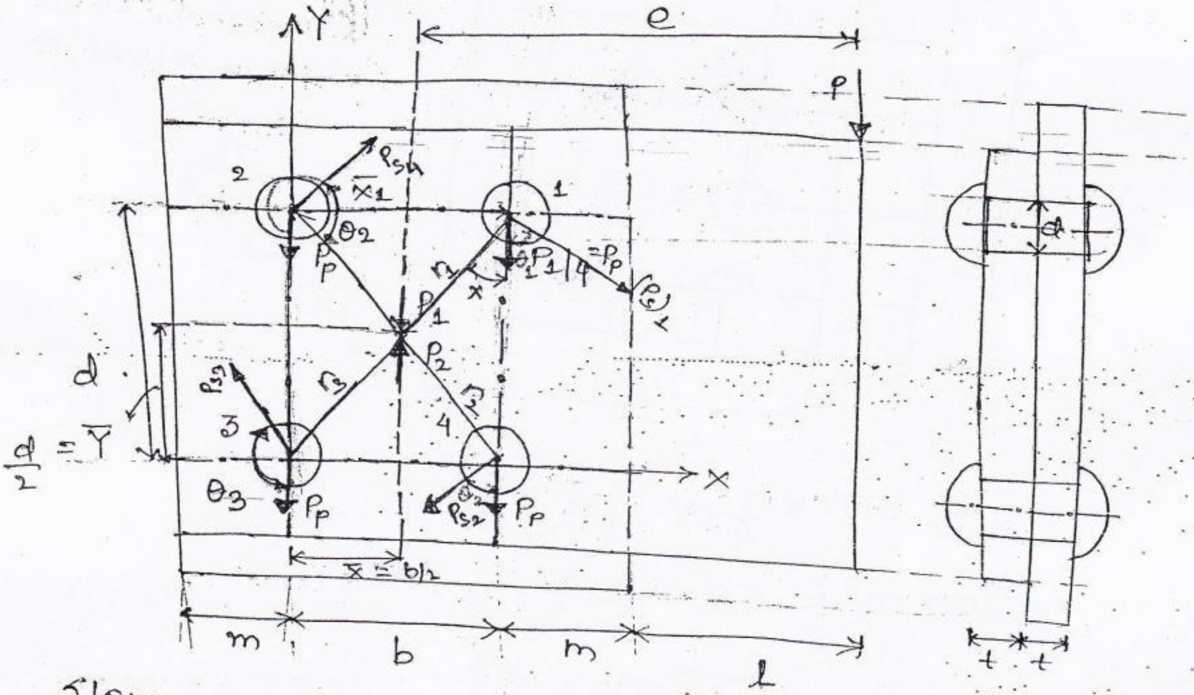
$\Rightarrow 100\times10^3 = [120 - 3\times(20\times2)]\times10\times\sigma_t$

$\Rightarrow \boxed{\sigma_t = 185.18\ MPa}$

- $\boxed{\text{Bearing is crushing stress}}$

Design of rivet joint under eccentric loading

when line of action of the load is not coinciding with the centre of gravity of rivet then loads are not equally distributed.



Steps

Step 1

(1) Centre of gravity of rivets.

$\theta_1 = 90 - x$

$\tan x = \frac{b/2}{d/2}$

$P_{s_1}, P_{s_2}, P_{s_3}, P_{s_4} \rightarrow$ Secondary force

$$\bar{X} = \frac{A_1 \bar{X}_1 + A_2 \bar{X}_2 + \cdots + A_4 \bar{X}_4}{A_1 + A_2 + A_3 + A_4}$$

$$\bar{X} = \frac{\bar{X}_1 + \bar{X}_2 + \bar{X}_3 + \bar{X}_4}{4}$$

$\bar{X}_1$ = Distance of centroid of first rivet from Y-axis.

$\bar{X}$ = centre of gravity

$\bar{X}_1 = b, \ \bar{X}_2 = 0, \ \bar{X}_3 = 0, \ \bar{X}_4 = b.$

$$\therefore \bar{X} = \frac{2b}{4} = \frac{b}{2}.$$

$\bar{Y}$ = Distance of centroid of rivet from x-axis.

$$\bar{Y} = \frac{\bar{Y}_1 + \bar{Y}_2 + \bar{Y}_3 + \bar{Y}_4}{4} = \frac{d+d+0+0}{4} = \frac{d}{2}.$$

Step 2 :-

Introduce two dummy loads P_1 and P_2 through the C.G of group of rivets as shown in the figure

$$P_1 = P_2 = P.$$

P_1 and P_2 are always equal and parallel to the applied load.

Step-3 →

Distance of line of action of applied load from the line of action of P_1 & P_2.

$$\boxed{e = l + m + \frac{b}{2}}$$

Step 4 →

Effect of P_1

Effect of P_1 is to cause a primary shear force (P_p) of equal magnitude at each and every rivet as shown in the figure.

- where secondary shear force is maxm thus it is worst rivet. Primary shear force is equal everywhere.
- one shear and one bending stress due to T.S.L But here no clearance thus no bending thus only shear stress is considered

Action → Always in the dirn of the load.

Reaction → opp. to the dirn of load.

$$\boxed{P_p = \frac{P_1}{n} = \frac{P}{4}\ (\downarrow)}$$

Direction of secondary force

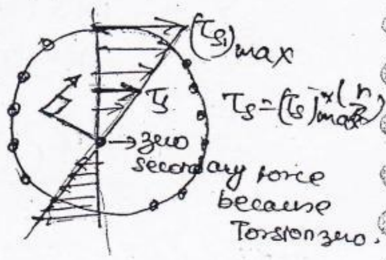
CW

$\downarrow P$

$P_2 \uparrow$

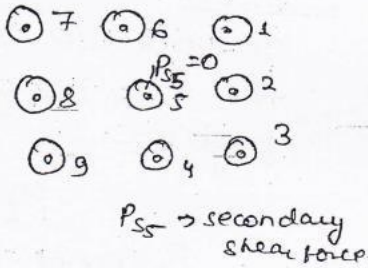
Step 5 :→

Effect of P & P_2



P and P_2 cause twisting couple.
Torsional shear stress ∝ r.
Angle of twist ∝ l.

Secondary shear force is equal for all the rivets equidistant from centre of gravity.



When secondary shear force and primary shear force are equal then the magnitude of resultant force depends only θ i.e, cos θ · less the θ more the result

Effect of P & P_2 causes a twisting moment w.r.t group of rivets. Due to this twisting moment all the rivets are subjected to secondary shear force (P_s).

Secondary shear force magnitude is directly proportional to r where 'r' is the distance b/w centre of gravity of group of rivets and the centroid of corresponding rivets.

Secondary shear force acts ⊥r to the line joining C.G of group of rivets and centroid of corresponding rivet. Secondary shear force is maxm at a rivet which is far away from the C.G of group of rivets i.e, where r is maxm.

Secondary shear force at all the rivets is equal in magnitude when all the rivets are equidistant from the C.G of group of rivets.

Step 6 :-

Calculation of r_1, r_2, r_3 and r_4.

$$r_1 = r_2 = r_3 = r_4 = \sqrt{\left(\frac{b}{2}\right)^2 + \left(\frac{d}{2}\right)^2} = \_\_\_\_$$

$$P_{s_1} = P_{s_2} = P_{s_3} = P_{s_4}$$

Step 7 :-

Calculation of $P_{s_1}, P_{s_2}, P_{s_3}$ & P_{s_4} :-

$$P_{s_1} r_1 + P_{s_2} r_2 + P_{s_3} r_3 + P_{s_4} r_4 = P \times e$$

$$\boxed{(P_{sn}) = P_{s_1}\left(\frac{r_n}{r_1}\right)} \quad (\because P_s \propto r)$$

$$\cancel{P_{s_1} r_1 + P_{s_2}\left(\frac{r_2}{r_1}\right) + P_{s_3}\left(\frac{r_3}{r_1}\right) + P_{s}}$$

$$P_{s_1} r_1 + P_{s_1}\left(\frac{r_2}{r_1}\right) r_2 + P_{s_1}\left(\frac{r_3}{r_1}\right) r_3 + P_{s_1}\left(\frac{r_4}{r_1}\right) r_4 = P \times e$$

$$\Rightarrow \boxed{\frac{P_{s_1}}{r_1}\left[r_1^2 + r_2^2 + r_3^2 + r_4^2\right] = P \times e}$$

$$P_{s_1} = \_\_\_\_ \text{ KN}$$

$$P_{sn} = P_{s_1}\left(\frac{r_n}{r_1}\right) = \_\_\_\_$$

Step 8

Finding resultant shear force

$$(\theta_1 = \theta_4) < (\theta_2 = \theta_3)$$

$$\boxed{\theta_{min} = \theta_1 \text{ or } \theta_4}$$

These are worst rivets as θ is min & or critical

(9) R_{max} (Resultant shear force on each rivet).

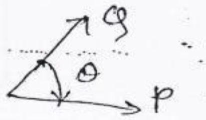
$$(R_1 = R_4) > (R_2 = R_3)$$

$$\left[\because P_{s_1} = P_{s_2} = P_{s_3} = P_{s_4} \quad (\theta_1 = \theta_4) < (\theta_2 = \theta_3)\right].$$

$$R_{max} = R_1 = R_4 = \sqrt{P_p^2 + (P_s)_1^2 + 2 P_p P_{s_1} \cos\theta_1}$$

$$= \_\_\_\_ \text{ KN}.$$

(10) Diameter of Rivets (d).



$R = \sqrt{P^2 + Q^2 + 2PQ\cos\theta}$

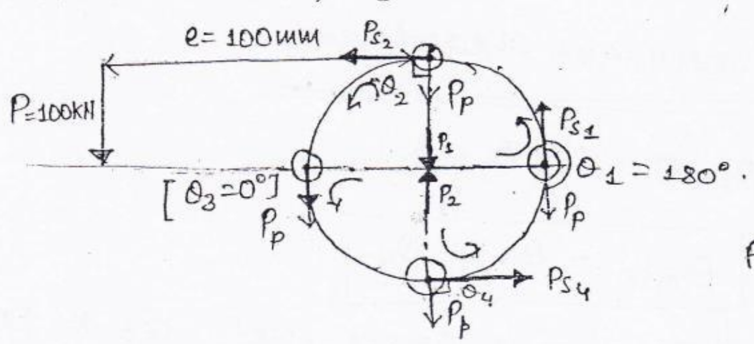
$$(\tau_{max})_{ind} \le \tau_{per}$$

$$\frac{R_{max}}{\left(\frac{\pi}{4} d^2\right)} \le \tau_{per}.$$

$$d \ge \_\_\_\_ \text{ mm}.$$

- Here, shear (Two shear stress acts torsional & shear stress).

(Q) For an eccentrically loaded riveted joint as shown in the fig. Determine
(i) Diameters of the rivets if permissible shear stress, $\tau_s = 60$ MPa.



$\theta \rightarrow$ angle b/w primary & secondary shear forces.

- worst rivet is the rivet which is nearest to the applied load.

Distn of Secondary Shear force is taken w.r.t to distance from the centre i.e, r.

(a) $P_p = \frac{P}{4} = \frac{100}{4} = 25\,kN$.

(b) $r_1 = r_2 = r_3 = r_4 = 50\,mm$

$\therefore P_{s_1} = P_{s_2} = P_{s_3} = P_{s_4}$.

(c) $\frac{P_{s_1}}{r_1}\left[r_1^2 + r_2^2 + r_3^2 + r_4^2\right) = P \cdot e$

$\Rightarrow \frac{P_{s_1}}{\cancel{r_1}} \times 4 r_1^{\cancel{2}} = 100 \times 100$

$\Rightarrow P_{s_1} \times 4 \times 50 = 10^4$

$\Rightarrow \boxed{P_{s_1} = 50\,kN}$

(d) $(\theta_3 = 0^\circ) < (\theta_2 = \theta_4 = 90^\circ) < (\theta_1 = 180^\circ)$

$R_3 > (R_2 = R_4) > R_1$.

(e) $R_{max} = R_3 = \sqrt{P_p^2 + P_{s_3}^2 + 2P_p P_{s_3}}$

$= \sqrt{(P_p + P_{s_3})^2} = \cancel{75\,kN}\; P_p + P_{s_3} = 25 + 50 = 75\,kN$.

(f) $(\tau_{max})_{ind} \leq \tau_{per}$

$\frac{4 R_{max}}{\pi d^2} \leq 60$

$\Rightarrow \frac{4 \times 75 \times 10^3}{\pi d^2} \leq 60$

$\Rightarrow d \geq \cancel{40\,m}\; 39.89\,mm$

$\Rightarrow d \geq 40\,mm$.

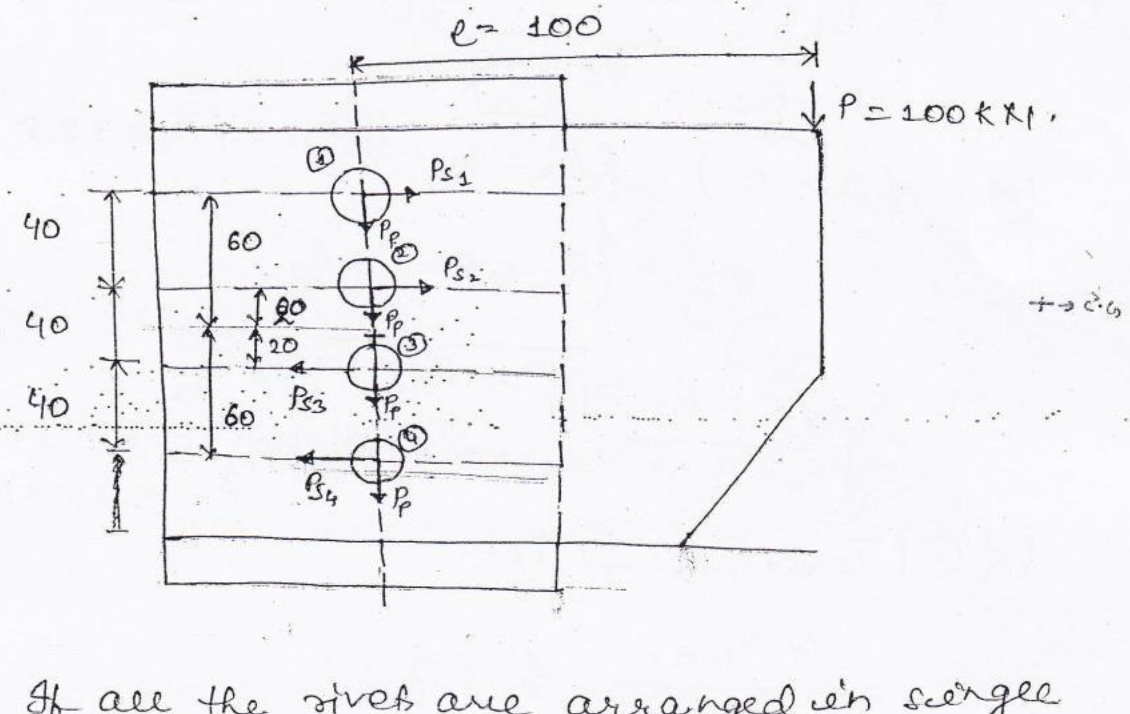
- Most safest rivet is the rivet which is far away.
- If all the rivets are equidistant from the C.G of group of rivet then the worst rivets are those rivets which are nearer to line of action of the load.

⑧ For an eccentrically loaded riveted joint as shown in the figure. Determine

(i) worst rivets

(ii) Maxm shear developed in the rivets if diameters of the rivet is 40mm.

(iii) Resultant shear force at each and every rivet.



If all the rivets are arranged in single vertical row then the worst rivet is the rivet which is far away from C.G.

worst rivet

(a) only 1.

(b) 2 & 3

(c) only 1 & 4.

(d) All of the rivets.

$(R_1 = R_4) > (R_2 = R_3)$

$[\because \theta_1 = \theta_2 = \theta_3 = \theta_4 = 90°]$

$(P_{s_1} = P_{s_4}) > (P_{s_2} = P_{s_3})$

(1) $P_p = \frac{P}{4} = 25\text{ KN}$.

(2) $r_1 = r_4 = 60\text{ mm}$.

$r_2 = r_3 = 20\text{mm}$

(3) $\frac{P_{s_1}}{r_1}\left[2r_1^2 + 2r_2^2\right] = P \times e$.

$\frac{P_{s_1}}{60}\left[2\times60^2 + 2\times20^2\right] = 100\times100$.

$P_{s_1} = 75\text{ KN} = P_{s_4}$

$P_{s_2} = P_{s_3} = P_{s_1}\left[\frac{r_2}{r_1}\right] = 25\text{ KN}$.

(4) $\theta_1 = \theta_2 = \theta_3 = \theta_4 = 90°$.

(5) $R_1 = R_4 = \sqrt{P_{s_1}^2 + P_p^2} = 79.05\text{ KN}$.

$R_2 = R_3 = \sqrt{P_{s_2}^2 + P_p^2} = 35.35\text{ KN}$.

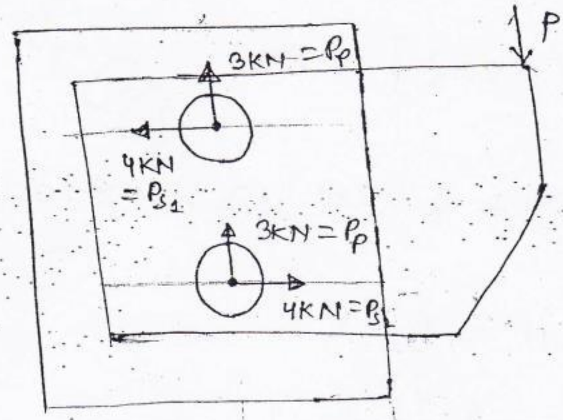
(6) $\tau_{max} = \tau_1 = \tau_4 = \frac{4R_1}{\pi d^2}$

$= \frac{4\times79.05\times10^3}{\pi\times(40)^2}$

$= 62.9\text{ MPa}$.

- when all the rivets are arranged in a single vertical row then the worst rivet are those rivets which are far away from the C.G of group of rivets.

(8) For an eccentrically loaded riveted joint as shown in the figure. Determine which of the following statements are correct, if area of all the rivets is 100 mm².



Statement

(1) Max^m shear stress developed in the rivet is 50 MPa.

(2) load acting on the joint is 6 KN.

(3) Eccentricity for the load is 100 mm.

(a) only 2nd statement is correct.

(b) 2nd & 3rd " are ".

(c) (1) & (2) are ".

centroid is exactly at the centre, ~~there~~ as secondary shear force are equal.

$$P_p = \frac{P}{n}$$

$$\Rightarrow P = nP_p = 6\,KN.$$

$$R_1 = R_2 = \sqrt{3^2+4^2} = 5\,KN.$$

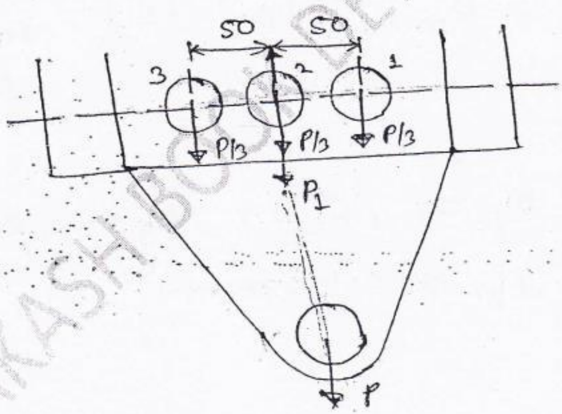
$$\tau_{max} = \tau_1 = \tau_2 = \frac{5000}{100} = 50\,MPa.$$

$$\frac{P_{s_1}}{r_1}(2r_1^2) = P \times e$$

$$\Rightarrow 4 \times 2 \times r_1 = 6 \times e$$

$$\Rightarrow \boxed{e = \frac{4}{3} r_1}$$

(8) Determine worst rivet, expression for the diameter of the rivets if permissible shear stress is equal to τ MPa



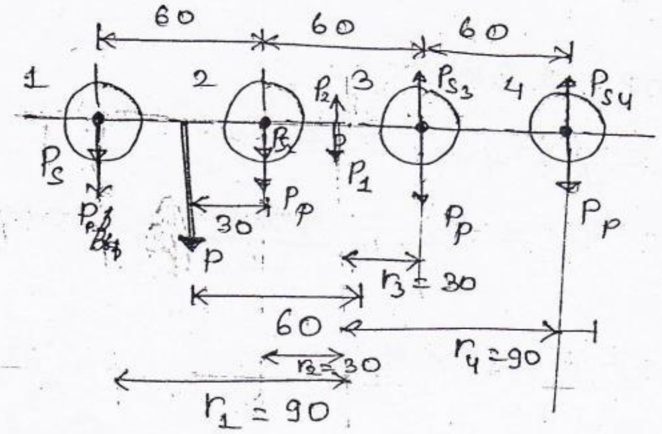
Here, all the rivets are worst rivet as the load is not eccentric.

$$R_1 = R_2 = R_3 = P_p = P/3 \left[\because \begin{array}{l} P_s = 0 \\ e = 0 \end{array} \right]$$

$$\tau_{max} = \tau_1 = \tau_2 = \tau_3 = \frac{4P_p}{\pi d^2} = \frac{4P}{3\pi d^2} \leq \tau$$

$$\Rightarrow \boxed{d \geq \sqrt{\frac{4P}{3\pi\tau}}}$$

(3) Find worst rivet :-



Sol

$$(r_1 = r_4 = 90) > (r_2 = r_3 = 30)$$

$$(P_{s_1} = P_{s_4}) > (P_{s_2} = P_{s_3})$$

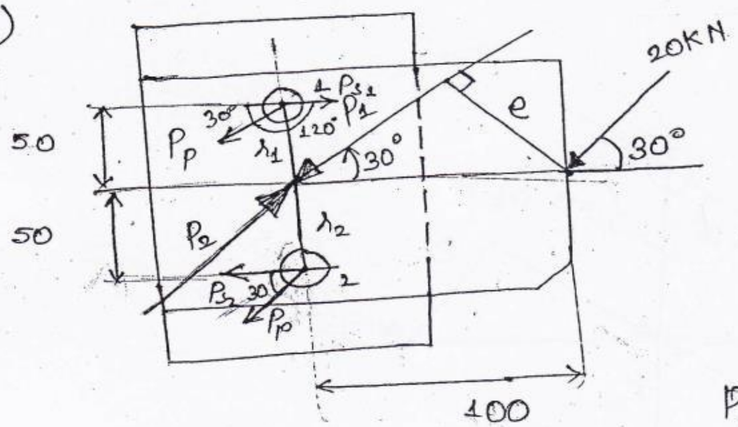
$$(\theta_1 = \theta_2 = 0^\circ) < (\theta_3 = \theta_4 = 180^\circ).$$

$$R_1 = R_{max} = P_p + P_{s_1} \left(\because r \text{ is } max \text{ \& } \theta \text{ is } min \right)$$

If $P = 50 KN$, $R_1 = ?$, $R_2 = ?$, $R_3 = ?$, $R_4 = ?$

$\tau_{max} = ?$ if $d = 30 mm$.

(Q)



$\sin 30^\circ = \frac{e}{100}$

$\Rightarrow \boxed{e = 50mm}$

$P_p = \frac{P}{2} = 10 KN.$

$r_1 = r_2 = 50mm.$

$\frac{P_{s_1}}{r_1}\left[2r_1^2\right] = 20 \times 50$

$\Rightarrow P_{s_1} \times 2 \times \cancel{50} = 20 \times \cancel{50}$

$\Rightarrow \boxed{P_{s_1} = 10 KN}$

$P_{s_1} = P_{s_2} = 10KN$

$\theta_1 = 150^\circ$

$\theta_2 = 30^\circ$

$R_2 > R_1$

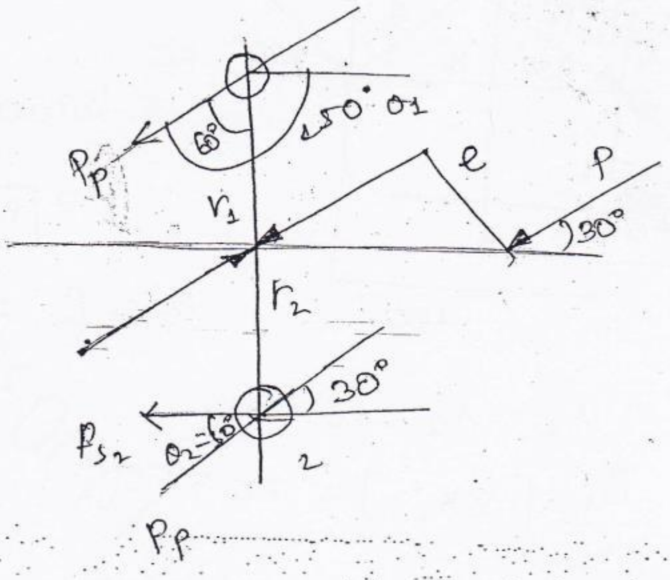
$R_{max} = R_2 = \sqrt{P_p^2 + 2P_p P_{s_2} \cos\theta_2}$

$= 19.3 \, KN.$

$\tau_{max} \le \tau_{per}.$

$\frac{4R_{max}}{\pi d^2} \le 100$

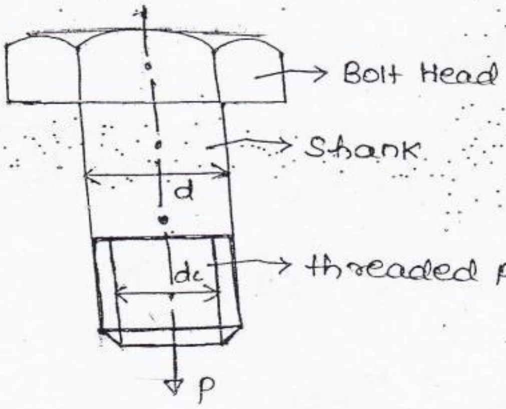
$\Rightarrow d \ge 49.58 \text{ mm}.$



13/01/14

Design of Bolted joints under Eccentric loading

Bolt designation



Critical portion is threaded portion, stress induced is maxm.

d_c = Minor (or) core Dia. (used in stress analysis (or) design calculation).

d = MAJOR (or) Nominal dia. (used in bolt designation).

$$\boxed{d = \frac{d_c}{0.84}}$$

- Metric thread → fastening purpose

$$(\sigma_{max})_{ind} \leq (\sigma_t)_{per}$$

$$\Rightarrow \frac{P}{\frac{\pi}{4} d_c^2} \leq (\sigma_t)_{per}$$

$$d_c \geq \_\_\_\_ \text{ mm.}$$

$$\boxed{d = \frac{d_c}{0.84}}$$

Designation of bolt

M 20 × 2 ⇒ Pitch] → fine pitch.

(or) M 20] → Coarse pitch.

M ↓ Thread profile

20 → Major or nominal dia.

M indicates → Metric thread.

fine pitches

Pitch / dia	2	4	6	8	10
20	✓	✓	✓	✓	✓
24	✓	—	✓	✓	—

10 → coarse pitch

8 → Coarse pitch for dia of 24 mm.

- For Automobile application fine pitch is better as vibrations are more (mainly in impact load cases)
- ~~Core~~ For static loading applications, where vibrations are less Coarse pitch are used.

(or)

Fine pitch are used in the applications where joint is subjected to dynamic load (i.e, jerky operation).

Applications are mainly in automobile and aircraft applications components.

Coarse pitch :- used in the applications where vibrations are less.

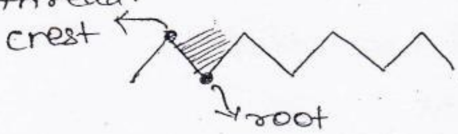
SQ 20 x 2

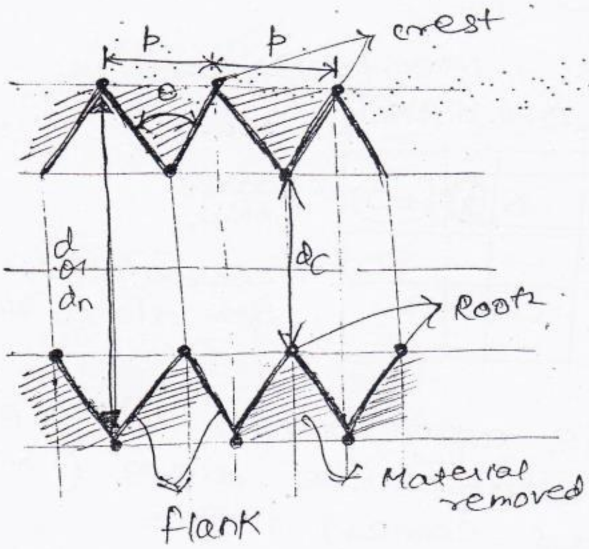
square thread → Mainly used in power transmission applications.

ACME 20x20

Major diameter

Dia. of an imaginary circle touching the crest of an external thread.s roots of internal thread.





Blue line represent cut portion or the metal removal portion.

$p \rightarrow$ pitch

$d_c \rightarrow$ Minor dia.

$\theta \rightarrow$ thread angle

In case of Metric thread → 60°

Pitch

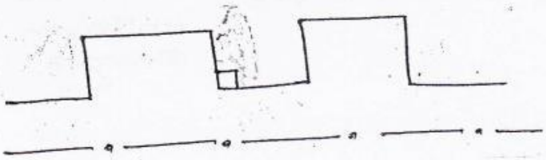
Distance measured b/w the parallel ~~threads~~ to the axis of the thread b/w corresponding pts. or adjacent flank.

Thread angle

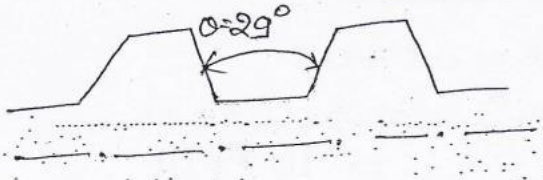
Angle b/w two adjacent flank.

Core dia - or Minor dia
Dia of an imaginary circle roots of external thread and root crest of an internal thread.

Square thread → thread angle → 0°.

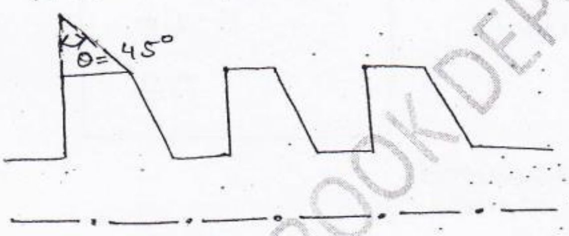


Acme thread



used in lathe & lead screw m/c.

Buttress thread



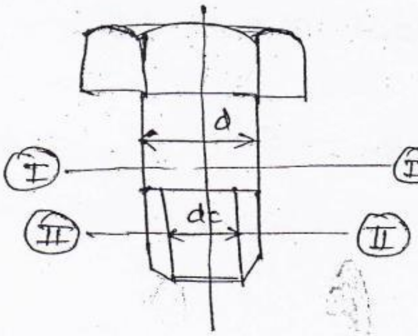
used for power & torque transmission in one dirn.

- If lead = pitch ($l = p$) then it is called single start thread.
  - If $l = np \Rightarrow$ Multi start thread.

- Axial distance covered by screw, thread in single rotation is called lead.

- Multi start threads are used when more axial distance is to be covered.

- critical c/s
  A cross-section passing through the roots is critical cross-section in bolt threaded portion.



$U \to$ strain energy of bolt of non-uniform strength.

$$\sigma_{I-I} = \frac{4P}{\pi d^2}$$

$$\sigma_{II-II} = \frac{4P}{\pi d_c^2}$$

$\sigma_{II-II} > \sigma_{I-I} \Rightarrow$ Bolt of uniform strength.

$U = U_{shank} + U_{T.P}$

$$U = \frac{\sigma^2}{2E} \times A \times L$$

$U_{T.P} > U_{shank}$

$\boxed{U = U_1 + U_2}$

- $\sigma_{SHANK} = \sigma_{\text{Threaded portion}}$

$\Downarrow$

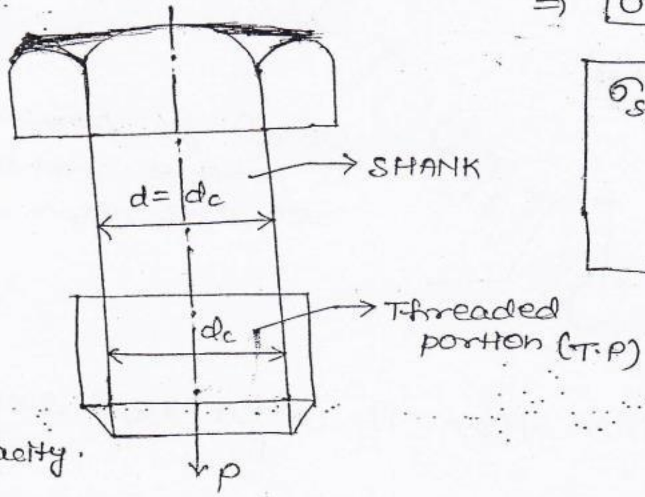
Bolt of uniform strength.

Method I

$U^* = U_1^* + U_2^*$

$U_2^* = U_2 \Rightarrow U_1^* \geq U_1$

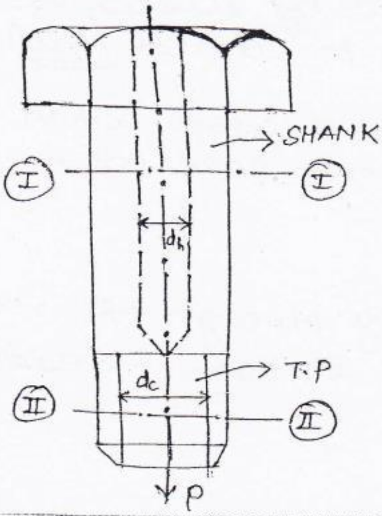
$\Rightarrow \boxed{U^* > U}$



$$\boxed{\begin{aligned}\sigma_{SHAK} &= \sigma_{T.P} \\ &= \frac{4P}{\pi d_c^2}\end{aligned}}$$

This bolt has more energy absorbing capacity.

Method 2 (Drilling hole)



For bolt of uniform strength

$$\sigma_{I-I} = \sigma_{II-II}$$

$$\frac{P}{\frac{\pi}{4}(d^2 - d_h^2)} = \frac{P}{\frac{\pi}{4} d_c^2}$$

$$d^2 - d_h^2 = d_c^2$$

$$** \boxed{d_h = \sqrt{d^2 - d_c^2}}$$

$$d_c = 0.84\,d$$

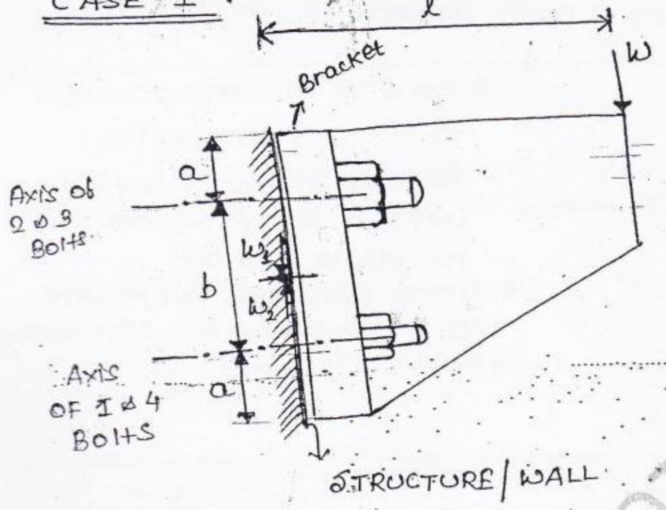
$$d_h = \sqrt{d^2 - (0.84d)^2}$$

$$\boxed{d_h = 0.54\,d}$$

Bolt of uniform strength is used in the applications where bolt is subjected to impact loading because its strain energy (i.e, Resilience) ~~of~~ is more than the strain energy of bolt of non-uniform strength.

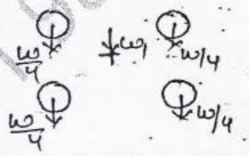
Bolt under eccentric loading

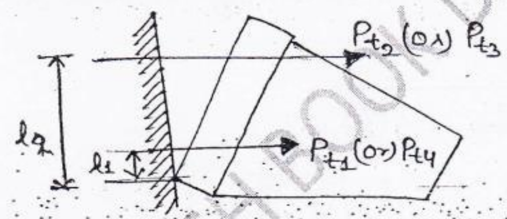
CASE I :- (Primary tensile & secondary tensile & shear)
(All the practical application comes under this)
(Bolt Joint) (T.S.L)



[load is acting ⊥r to axis of Bolts (or) load is acting in a plane which is parallel to plane of bolts]

Here bolts are subjected to shear and tensile stresses





Step: 1

Introduce two equal and opposite forces through the C.G of group of bolts as shown in the fig.

$$W_1 = W_2 = W$$

(2) $e = l$. (3) Effect of W_1 is to cause a shear force of equal magnitude at each and every bolt.

$$\boxed{P_s = \frac{W_1}{n} = \frac{W}{4}} \quad \text{— (1)}$$

$$\tau_s = \frac{P_s}{A_s} = \frac{4P_s}{\pi d_c^2} = \frac{W}{\pi d_c^2} \Rightarrow \tau_s = \frac{W}{\pi d_c^2} = \frac{X}{d_c^2} \quad \text{— (I)}$$

(4) Effect of W & W_2 : causes a moment of $W \times e$ in cw dirn w.r.t bracket, due to this moment as the bracket is tilted about the bottom edge as shown in the figure. Bolts are subjected to tensile forces (P_t). Tensile force developed at every bolt is directly proportional to l where l is the distance b/w axis of the bolt and tilting edge. Hence tensile force is maxm at a bolt which is far away from the tilting edge.

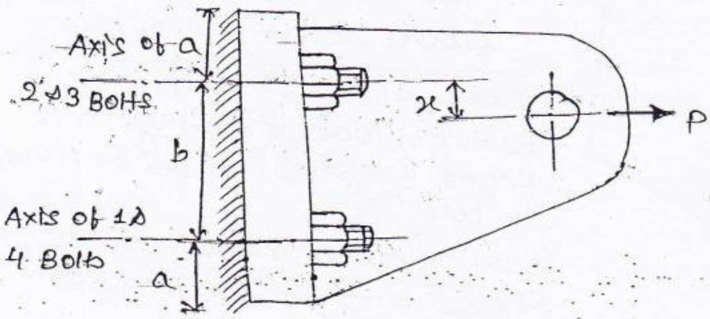
$M = w \times e = \_\_\_\_ \ N\text{-}mm.$ | (5) l_1, l_2, l_3, l_4 $(l_2 = l_3) > (l_4 = l_1)$

$\Rightarrow (P_{t2} = P_{t3}) = (P_{t1} = P_{t4})$ [$\because P_t \propto l$] $\Rightarrow (P_t)_{max} = P_{t2} = P_{t3}$

(6) $(P_t)_{max}$:- $\frac{P_{t1}}{l_1}(2l_1^2 + 2l_2^2) = w \times e \Rightarrow P_{t1} = \_\_\_ \Rightarrow (P_t)_{max} = (P_{t1})\left(\frac{l_2 (or)\, l_3}{l_1}\right) = \_\_\_$

$$(\sigma_t)_{max} = \frac{4(P_t)_{max}}{\pi d_c^2} = \frac{Y}{d_c^2} \quad \longleftarrow (2)$$

<u>Case II</u> :- (Tensile & Tensile) (Primary, secondary) [Butt Joint] [Eccentric axial load].
Here dummy load acts parallel to the axis of bolt.



- load is acting parallel to the axis of bolts (or) load is acting in a plane which is perpendicular to plane of bolts.
- Here bolts are subjected to primary and secondary tensile stresses.

Case 1 → continued ---

(7) <u>Design of Bolts</u>

Bolts are designed by using either MSST or MDET because they are subjected to combined stresses and they are made up of ductile material.

MSST,

$$(\sigma_t)_{per} = \frac{S_{yt}}{N} = \sqrt{(\sigma_t)^2_{max} + 4(\tau_s)^2}$$

$$\tau_{per} = \frac{S_{ys}}{N} = \frac{S_{yt}}{\sqrt{3}} \quad \frac{S_{yt}}{2N} = \frac{1}{2}\sqrt{(\sigma_t)^2_{max} + 4\tau_s^2}$$

$$\tau_{per} = \frac{S_{ys}}{N} = \frac{1}{2}\sqrt{\left(\frac{Y}{d_c^2}\right)^2 + 4\left(\frac{X}{d_c^2}\right)^2}$$

$$\Rightarrow d_c = \_\_\_\_ \ mm \Rightarrow \boxed{d_n = d_c/0.84}$$

MDET,

$$\tau_{per} = \frac{S_{ys}}{N} = \frac{S_{yt}}{\sqrt{3}N} = \frac{1}{\sqrt{3}}\sqrt{(\sigma_t)^2_{max} + 3\tau_s^2}$$

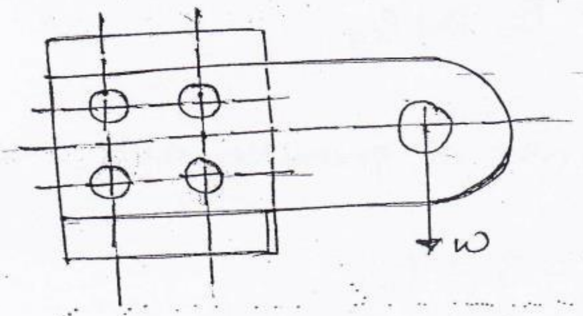
$$\tau_{per} = \frac{S_{ys}}{N} = \frac{1}{\sqrt{3}}\sqrt{\left(\frac{Y}{d_c^2}\right)^2 + 3\left(\frac{X}{d_c^2}\right)^2}$$

$$d_c = \_\_\_\_ \ mm$$

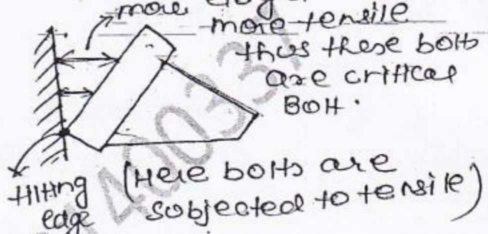
Also, $(\sigma_t)_{per} = \frac{S_{yt}}{N} = \sqrt{(\sigma_t)^2_{max} + 3\tau_s^2}$

$$\boxed{d_n = d_c/0.84}$$

Case-III (lap joint)



Direct of the case 1 moment gives the tilting dirn.



- lap joint
- load is acting in a plane which is away from the plane of bolts.
- Bolts are subjected to primary and secondary shear stresses.

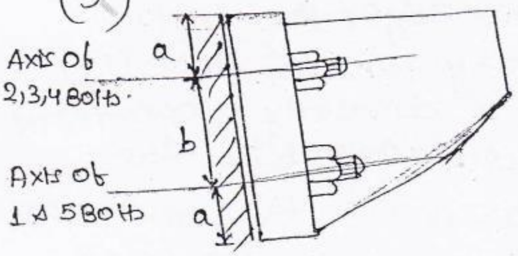
$$\tau_{max} \leq \tau_{per}$$

$$\frac{4 R_{max}}{\pi d_c^2} \leq \tau_{per} \Rightarrow d_c \geq \_\_ \text{ mm}$$

$$\boxed{d_n = \frac{d_c}{0.84}}$$

** The bolts which are far away from the tilting edge is the worst bolt.

(Q)

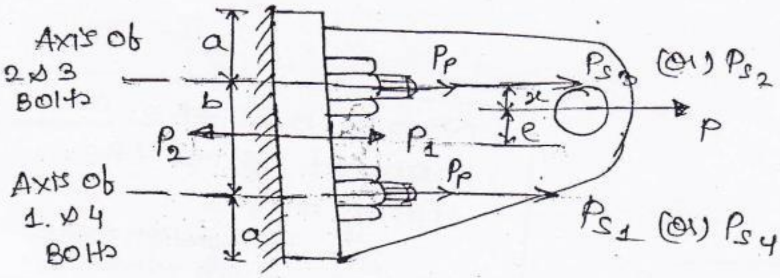


$w = 20\text{ kN}$, $n = 5$, $a = 50\text{ mm}$,
$b = 200\text{ mm}$, $l = 200\text{ mm}$,
$S_{ys} = 150\text{ MPa}$,
$N = 4$

MSST,
$d_c = 12.72$ mm
$d = 15.1$ mm

M16 Bolt.

Case II



(1) Determine centre of gravity and introduce dummy load.

$$P = P_1 = P_2 .$$

(2) $e = \frac{b}{2} - x$

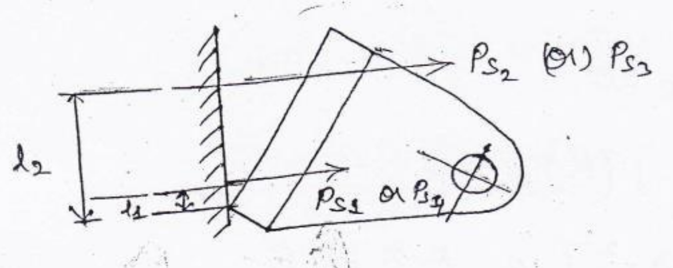
(3) **Effect of P_1**

Effect of P_1 is to cause a primary tensile force of equal magnitude at each and every bolt as shown in the figure.

$$P_p = \frac{P_1}{n} = \frac{P}{4} \quad \text{— (I)}$$

(4) **Effect of P & P_2**

P & P_2 causes a moment of $(P \times e)$ w.r.t bracket due to this moment as the bracket is tilted about ~~the bottom edges the~~ bolts are subjected to secondary tensile force (P_s). Secondary tensile force is directly proportional to l, where l is the distance b/w the tilting edge and the axis of the correspon-ding edge. Hence, secondary tensile force is maxm at a bolt which is far away from the tilting edge.



$$M = P \times e = \_\_\_\_ \text{ N-mm.}$$

(6) $(l_2 = l_3 = a+b) > (l_1 = l_4 = a)$

$$\left[P_{s_2} = P_{s_3} = (P_s)_{max}\right] > (P_{s_1} = P_{s_4})$$

(7) $\frac{P_{s_1}}{l_1}\left[2l_1^2 + 2l_2^2\right] = P \times e$

$$P_{s_1} = \_\_\_\_$$

$$(P_s)_{max} = (P_{s_1})\left[\frac{l_2 \text{ (or) } l_3}{l_1}\right].$$

(8) $R_{max} = R_2 = R_3 = P_p + P_s = \_\_\_\_ \text{ kN.}$

(9) <u>Dia. of bolt</u>

$$(\sigma_t)_{max} \leq (\sigma_t)_{per}$$

$$\frac{4 R_{max}}{\pi d_c^2} \leq (\sigma_t)_{per}$$

$$\Rightarrow d_c \geq \_\_\_\_ \text{ mm.}$$

$$\Rightarrow d_n = d_c / 0.84 = \_\_\_\_ \text{ mm.}$$

W.B (ch-5)
(14) All the rivets are arranged in single vertical row. worst Rivets A & C.
(a) $\sigma_A > \sigma_C$. (Only one option)

- For all objective paper directly use major dia for calculation, if ans will not match then use dc.

(14) $P_p = \frac{P}{3} = \frac{10}{3}$

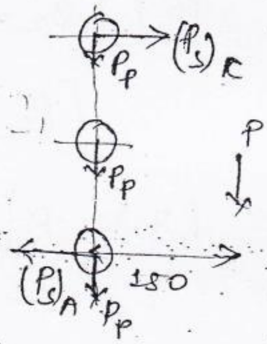
$r_A = r_C = 40$; $r_B = 0$

$(P_s)_A = (P_s)_{BC}$; $(P_s)_B = 0$.

$\frac{(P_s)_A}{r_A}\left[2r_A^2 + r_B^2\right] = F \times 150$

$\Rightarrow (P_s)_A \times 2 \times 40 = 10 \times 150$

$\Rightarrow (P_s)_A = 18.75\ KN$.



$R_B = P_p = \frac{10}{3}\ KN$.

$\tau_B = \frac{4R_B}{\pi d^2} = 4 \times \frac{10}{3} \times \frac{10^3}{\pi (10)^2}$

$= 42.4\ MPa$.

$R_A = R_C = \sqrt{P_p^2 + P_s^2} = \sqrt{\left(\frac{10}{3}\right)^2 + (18.75)^2}$

$= 19.04\ KN$.

$\tau_A = \tau_C = \frac{4R_A}{\pi d^2} = \frac{4 \times 19.4 \times 10^3}{\pi \times 10^2}$

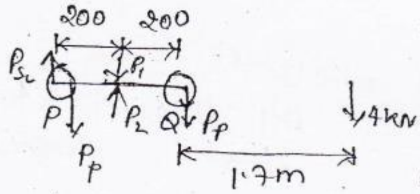
$= 247\ MPa$.

(19) (a) $\frac{4}{2} = 2$ (20) $\tau_{max} = \frac{4R_{max}}{\pi d^2}$

$R_B = R_{max} = \sqrt{4 + (20)^2 + 2 \times 4 \times 20}$

$= 2 + 20$

$= 22$

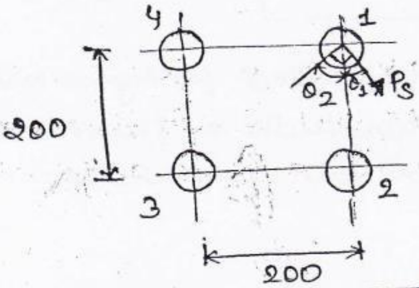


$\tau_B = \tau_{max} = \frac{4 \times 22 \times 10^3}{\pi \times (12)^2} = 194.52$.

$R_A = 20 - 2 = 18\ KN$.

$\tau_A = 18 \times 10^3 \times 4 / \pi \times 12^2 = 159.15\ MPa$.

(10) when r is same the nearest rivet is worst.



$(\tau_{max})_{1 \& 2}$

$$= \sqrt{\tau_p^2 + (\tau_s)^2 + 2\tau_p\tau_s\cos 45^\circ}$$

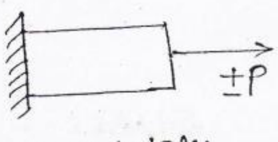
$$= 18.6\ MPa.$$

FATIGUE LOADING

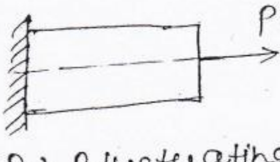
- Either magnitude or direction or magnitude & direction both changes.
- Fatigue failure is identified by the fracture. life of material under fatigue can be obtained using the no. of revolutions used.
- Fatigue strength is less than yield strength means there is no possibility of permanent deformation. occur both under static and fatigue load.

Definition of fatigue load

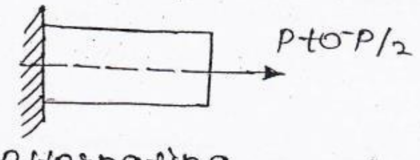
Fatigue loads are those loads whose magnitude and direction or direction or both magnitude changes w.r.t time and the loads are applied repeatedly w.r.t time.



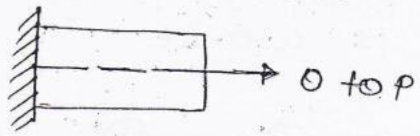
(1) Completely reversed fatigue loads.
(Here only dirn of the load changes)



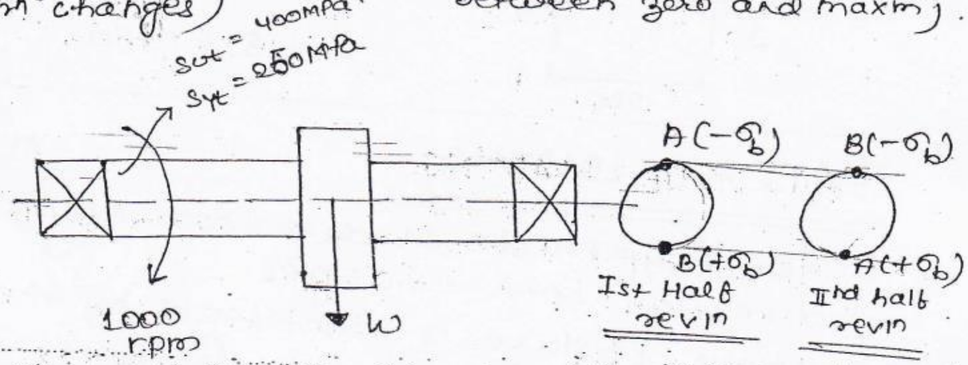
(2) Fluctuating fatigue loads.
(Here only magnitude of the load changes)



(3) Alternating fatigue load
(Both magnitude and dirn changes)

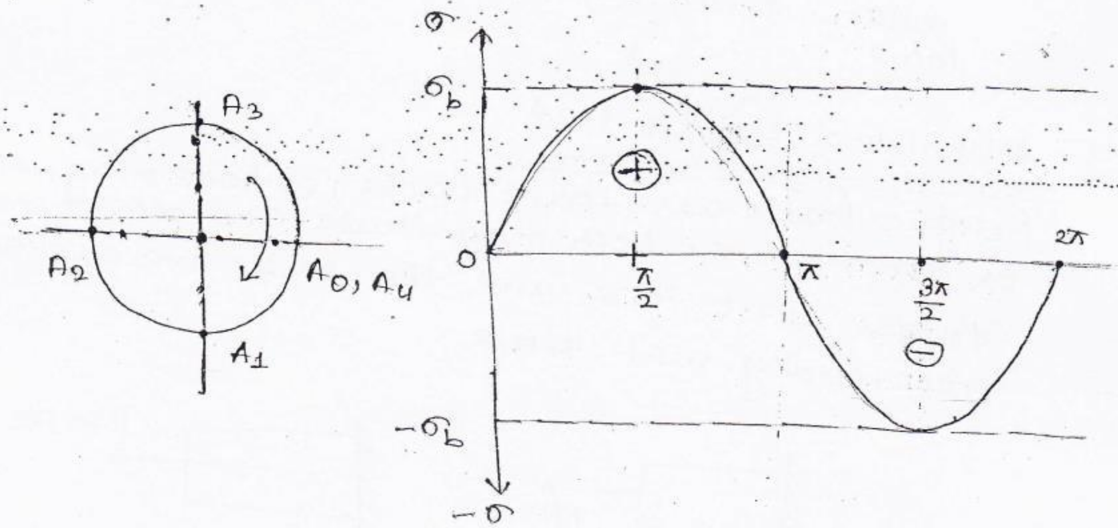


(4) Repeated fatigue load.
(Magnitude is fluctuating between zero and maxm).



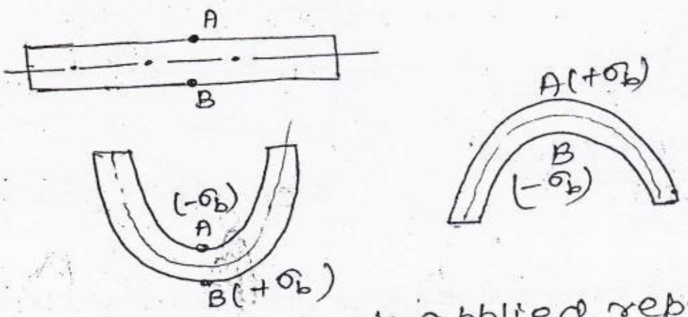
- w.r.t time → static load.
  w.r.t dirn → T.S.L
  w.r.t distribution → concentrated load.
  Sagging bending.

/ Fatigue failure not only occur under fatigue but also under static load.



Stress cycle of completely reversed stresses

- Always the fatigue failure occur before Syt in brittle material.



When same stress is applied repeatedly +

Fatigue is defined as a phenomena of failure or fracture of a component under fluctuating or fatigue stresses having a magnitude less than yield strength of the material (in case of ductile) or ultimate strength of the material (in case of brittle material)

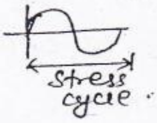
Fatigue is also be defined as the decreased resistance of the material under variable loading.

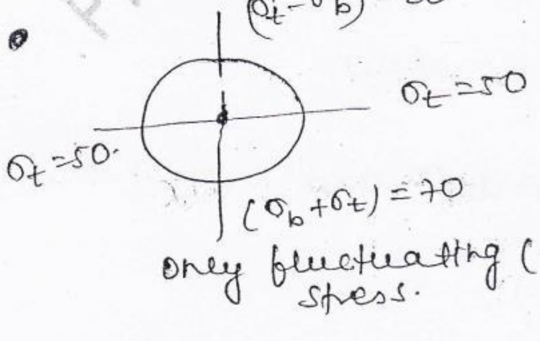
Condition for fracture:- $\boxed{\sigma_{ind} > S_{ut}}$ In case of static loading.

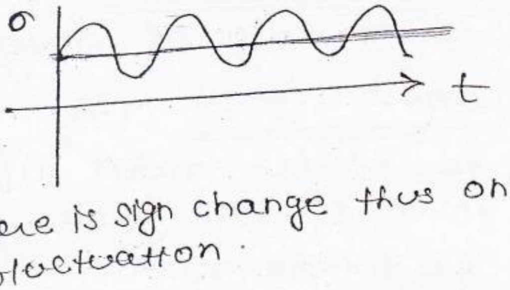
Fracture occurs before S_{yt} in case of ductile material

Stress cycle

Smallest portion of stress – time plot which is repeated periodically.



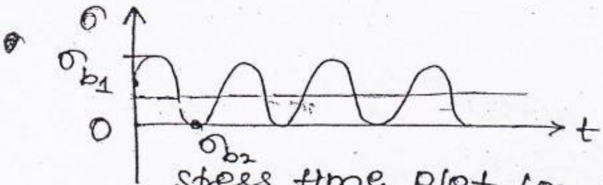




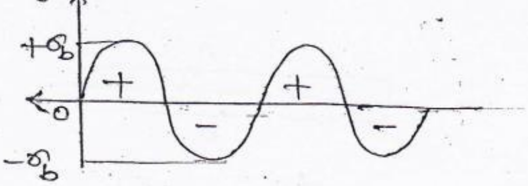
Only fluctuating stress. (there is sign change thus only fluctuation

unlike, different magnitude.

Alternating fatigue stress



stress time plot for Repeated stress



only dirn changes

- fatigue test is required to get the endurance limit.
- Static failure occur only under static load permanent deformation occur.

$\boxed{14/01/14}$

Max^m stress

It is the largest algebraic ~~ser~~ stress in a stress cycle.

Min^m stress

Smallest algebraic stress in a stress cycle.

Mean stress

Avg of Max^m and min^m stress in a stress cycle.

Range of stress

It is the diff. of max^m and min^m stress in a stress cycle.

Variable of stress

It is half of range of stress.

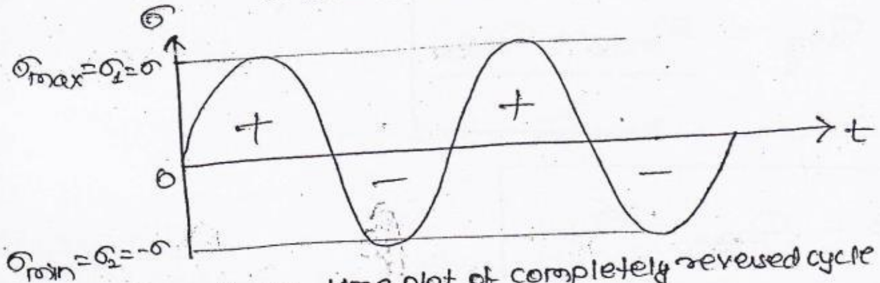
Stress ratio:- (R)

It is the ratio of min^m and max^m stress in a stress cycle.

Amplitude ratio - (A)

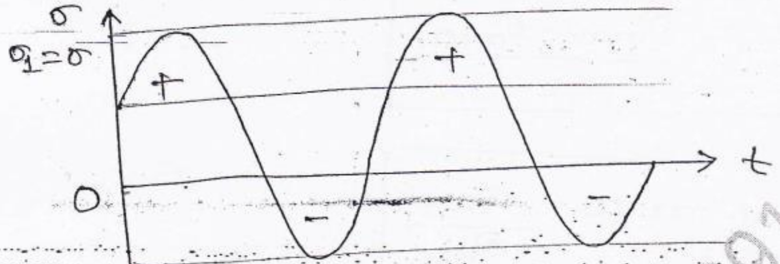
It is the ratio of variable stress and mean stress in a stress cycle.

- Fatigue is dirn dependent phenomena. Both magnitude & dirns are considered.



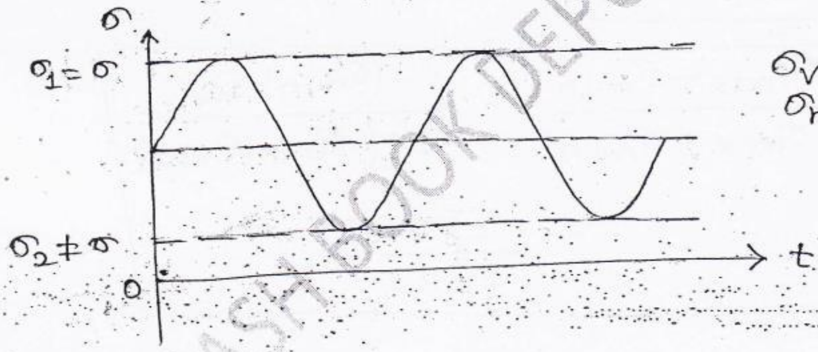
$\sigma_{max} = \sigma$
$\sigma_{min} = -\sigma$
$\sigma_m = 0$
$\sigma_r = 2\sigma = 2\sigma_{max}$
$\sigma_v = \sigma_{max}$
$R = -1$, $A = \infty$

Fig:- Stress-time plot of completely reversed cycle

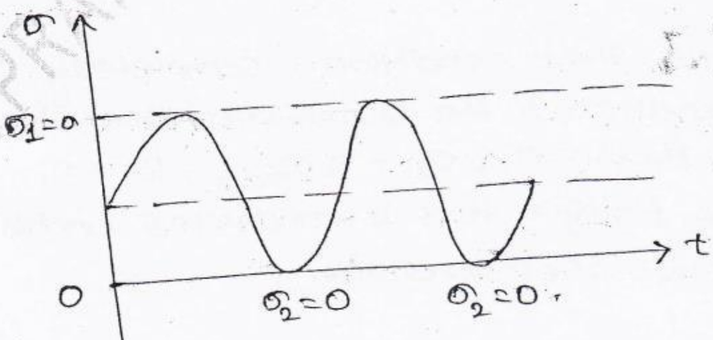


$\sigma_v \neq 0$
$\sigma_m \neq 0$.

Fig:- stress time plot of alternating stresses.



$\sigma_v \neq 0$
$\sigma_m \neq 0$.



$\sigma_{max} = \sigma$, $\sigma_{min} = 0$
$\sigma_m = \frac{\sigma_{max}}{2}$, $\sigma_r = \sigma_{max}$,
$\sigma_v = \frac{\sigma_{max}}{2}$,
$R = 0$, $A = 1$.

- Stress time plot of repeated stresses

$$\boxed{\sigma_m = \sigma_{avg} = \frac{\sigma_{max} + \sigma_{min}}{2}}$$

$$\boxed{\sigma_{range} = \sigma_{max} - \sigma_{min}}$$

$$\boxed{\sigma_v = \frac{\sigma_r}{2} = \frac{\sigma_{max} - \sigma_{min}}{2}}$$

$$\boxed{R = \text{Stress ratio} = \frac{\sigma_{min}}{\sigma_{max}}}$$

$$\boxed{A = \frac{\sigma_v}{\sigma_m} = \frac{1-R}{1+R}}$$

$$\boxed{A = \frac{\sigma_{max} - \sigma_{min}}{\sigma_{max} + \sigma_{min}} = \frac{1-(\sigma_{min}/\sigma_{max})}{1+(\sigma_{min}/\sigma_{max})} = \frac{1-R}{1+R}}$$

Statement

Out of all the above stress condition, completely reversed stress condition is the worst condition, hence (because mean stress $= 0$, $\sigma_r = 2\sigma_{max}$, $R = -1$, $A = \infty$) Hence fatigue test is conducted under completely reverse stress condition.

Endurance limit

Endurance limit is defined as the max^m value of completely reversed bending stress that a material can withstand for an infinite no of cycles without a fatigue failure (i.e, without a crack initiation)

Fatigue life is defined as the no. of revolutions that a component can undergo without a crack initiation.

The fatigue cracks are like to be initiated in the following three regions:-

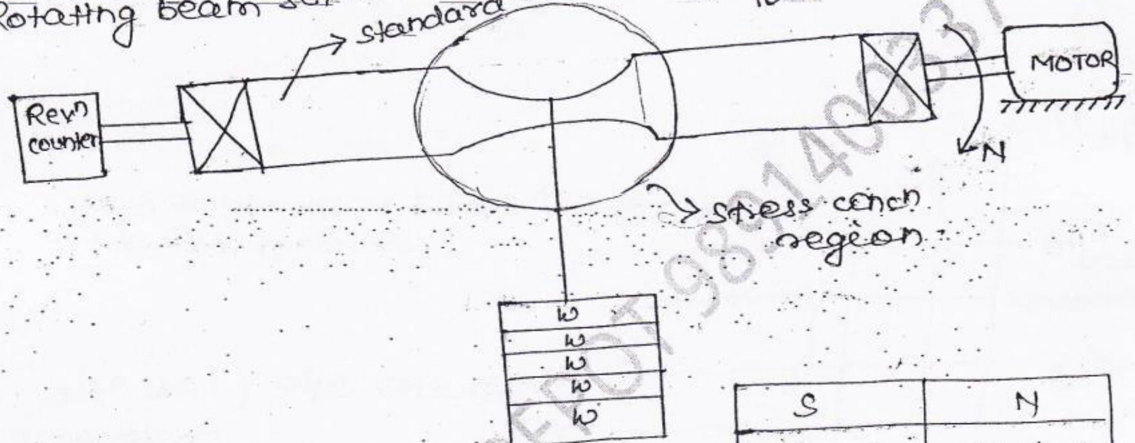
(i) Discontinuity or stress-concentration region

(ii) surface irregularities region.

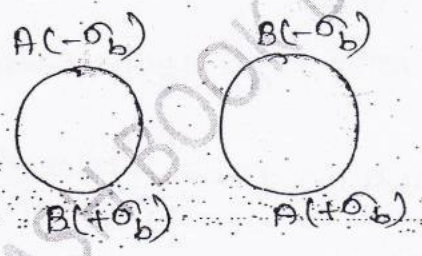
(iii) Defects

- This type of specimen is called standard specimen [d = 8 to 10 mm, SF = Mirror polished S.F, load = completely reversed bending]

Rotating beam set up.

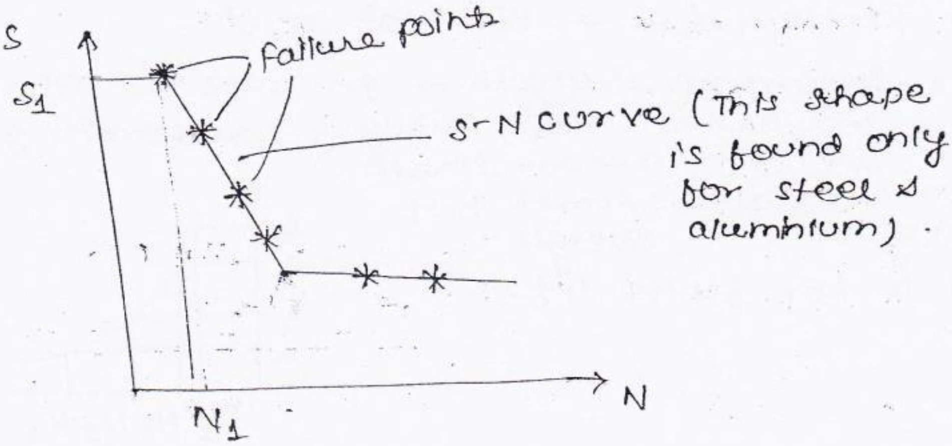


S		N	
S_1	DECREASES	N_1	INCREASES
S_2		N_2	
S_A		N_A	

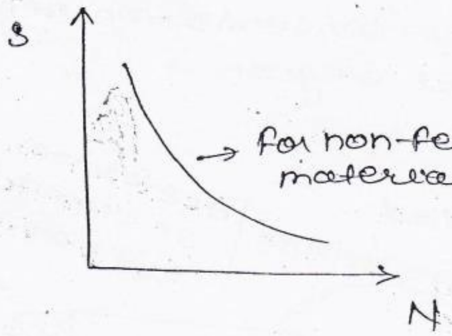


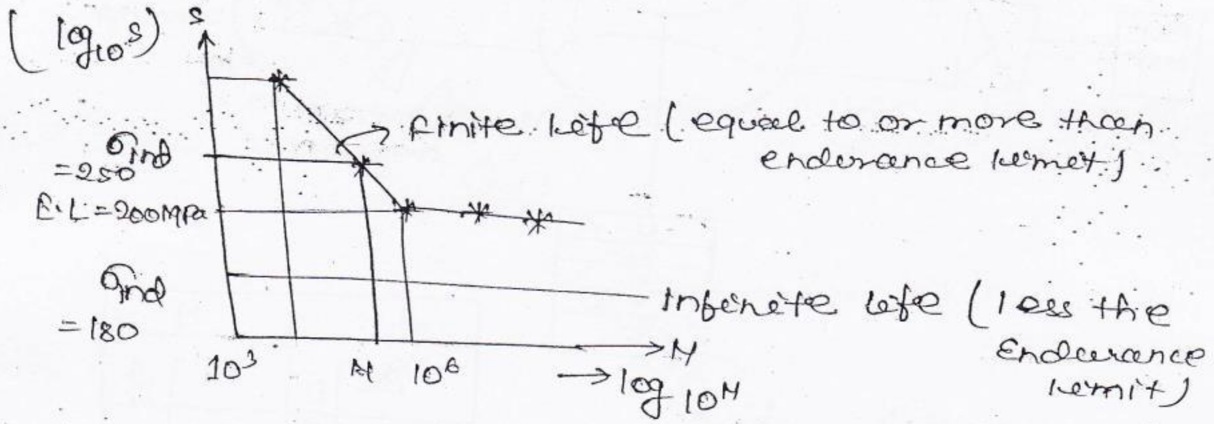
I^st Half revolution   II^nd Half revolution

- from S-N curve we get Endurance Limit



whenever the curve is become horizontal, the corresponding Y-coordinate is called endurance limit.



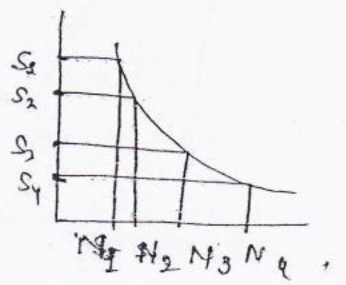


- Here, life is above 10^3 → High cycle fatigue

life b/w 0 → 1000 crack occur then it is low cycle fatigue.

- S-N-curve is valid for High cycle fatigue.
- So S-N curve is drawn on log curve x.
- finite life → B/w 10^3 to 10^6.
- Endurance strength → In case of non-ferrous material. Here, every y-coordinate give on specific x-coordinate.

No endurance limit but only " strength,
It is for finite life.



Endurance strength

Max^m value of completely reversed bending stress that a material can endure or withstand for a finite no. of cycle without a fatigue failure.

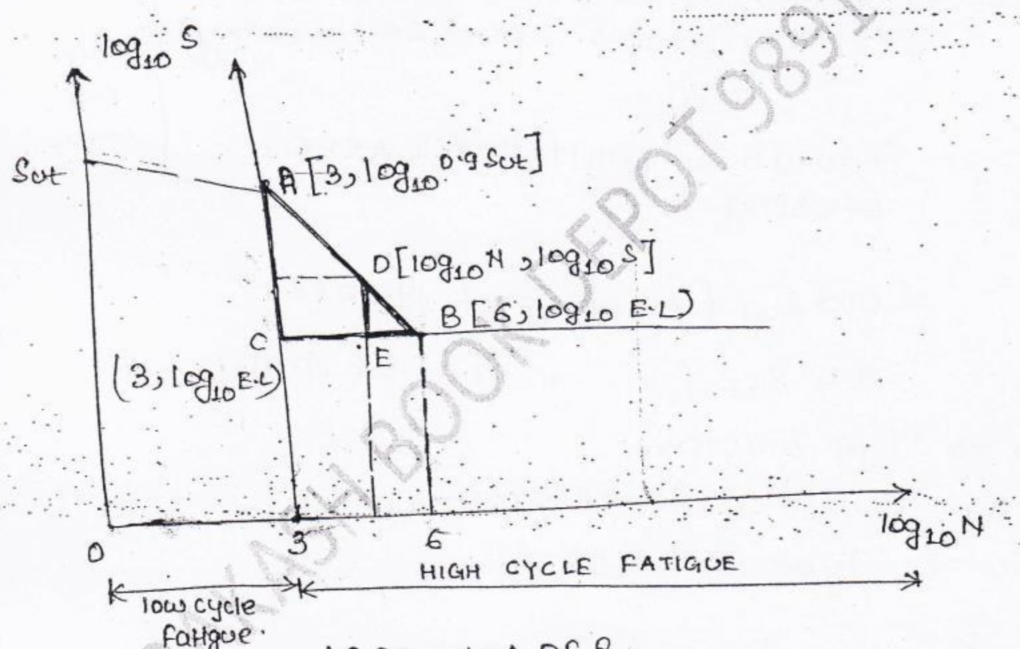
Endurance strength is represented by magnitude of stress followed by corresponding no. of cycles.

Endurance strength (E.S),

$$E.S = S_1 \text{ MPa at } N_1 \text{ cycles.}$$

$$E.S = S_2 \text{ MPa at } N_2 \text{ "}$$

$$E.S = S_3 \text{ " " } N_3 \text{ "}$$



$\Delta ABC \sim \Delta DEB$,

- log graph is considered only in high cycle fatigue.

$$\frac{AC}{DE} = \frac{CB}{EB}$$

$$\Rightarrow \frac{[\log_{10}(0.9 S_{ut}) - \log_{10}(E.L)]}{[\log_{10} S - \log_{10}(E.L)]} = \frac{6-3}{6 - \log_{10} N}$$

If S is given, $\log_{10} N = x$

$$\boxed{N = 10^x \text{ revolutions}}$$

when rpm = 0 mean S_{ut}. As the no. of revolutions increases, the chances of fatigue failure is more.

If N is given, $\log_{10} S = Y$

$$\Rightarrow \boxed{S = 10^Y \, MPa}$$

Eg:- S_{ut} = 200MPa (More chance of fatigue failure)
S_{ut} = 400MPa (Better as it can withstand upto 400MPa stress without any crack initiation.

Endurance limit of a Mechanical component :- (σ_e)

Size factor (K_a), surface finish factor (K_b), (K_c) Type of load.

$\sigma_e{}' \rightarrow$ endurance limit of a standard specimen (under reverse bending).
(or)
Endurance limit under reverse bending.
} from S-N curve.

= $0.5 S_{ut}$ (In case of steels)
$0.4 S_{ut}$ (" " " cast Iron).

$K_a \rightarrow$ size factor.
$K_b \rightarrow$ surface finish factor.
$K_c \rightarrow$ Type of load.

K_a, K_b, K_c lies b/w 0 & 1

$$\boxed{0 < (K_a, K_b, K_c) < 1}$$

- $K_a \rightarrow$ size factor $\Rightarrow$ size$\uparrow \Rightarrow$ Defects$\uparrow \Rightarrow$ cracks$\uparrow \Rightarrow$ E.L$\downarrow \Rightarrow K_a\downarrow$.
- $K_b \rightarrow$ S.F factor $\Rightarrow$ (Surface Roughness$\uparrow$ or Surface finish$\downarrow$) $\Rightarrow$ cracks$\uparrow \Rightarrow$ E.L$\downarrow \Rightarrow K_b\downarrow$
- K_c = load factor

In objective paper,
if K_a, K_b & K_c are not given then assume it 1

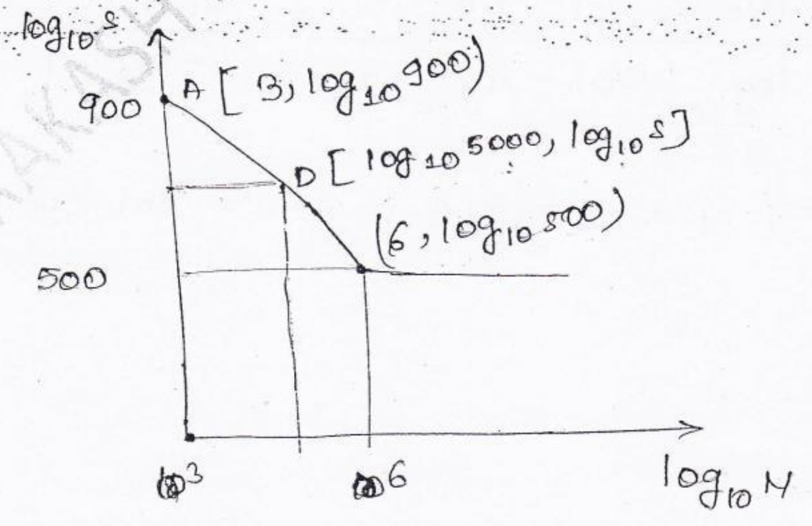
$K_c = 1$, for completely reversed cycle bending.
$K_c = 0.7$, " " reversed axial load.
$K_c = 0.6$, " " " torsion.

- chances of fatigue failure are g more under tension.
- " " " " " less " compression

- Fatigue strength is always less than static strength almost 50%.
- Chance of fatigue failure is more in torsion.
- " " " " less under bending.

(Q) A rotating beam test specimen when loaded to a stress of 900 MPa gives a life of 1000 load cycles. when loaded to a stress of 500 MPa gives a life of 10^6 cycles. If the required life of the component is 5000 cycles, find corresponding value of stress.

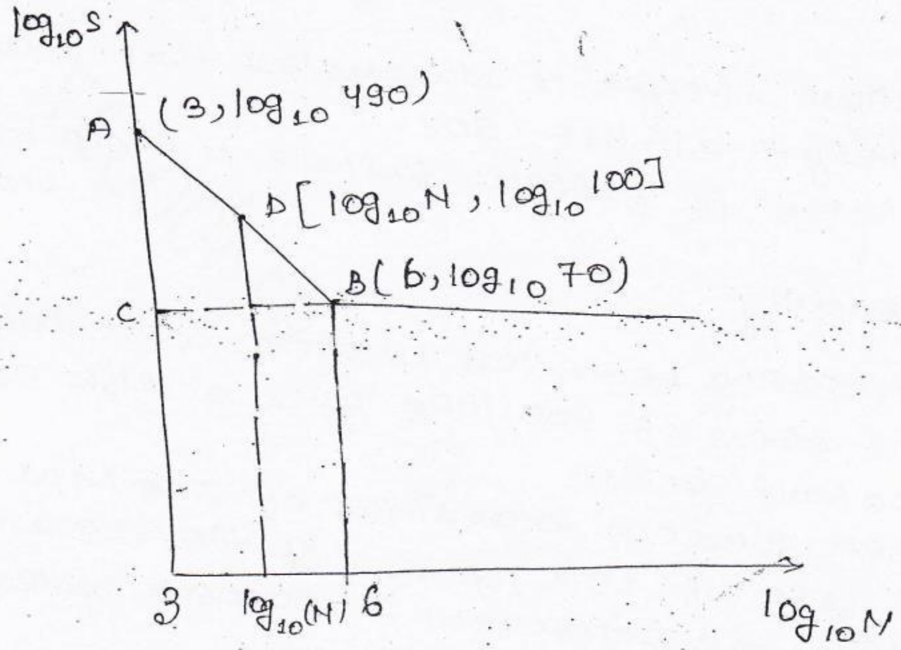
Sol



$$\frac{\log_{10}(900) - \log_{10}(500)}{\log_{10}(S) - \log_{10}(500)} = \frac{6-3}{6-\log_{10}50000}$$

$\Rightarrow \quad S = 645\text{ MPa}.$

W.B (9)

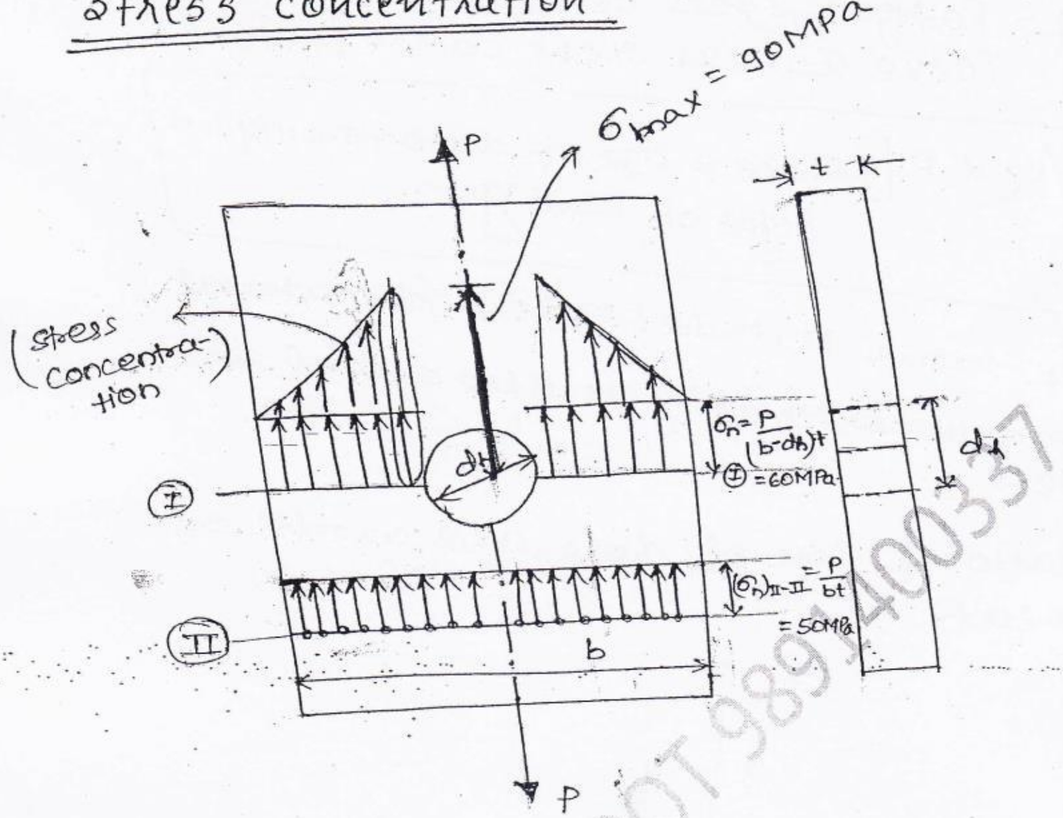


$-1.202 = \frac{1}{6-\log_{10}N}$

$$\frac{\log_{10}(490) - \log_{10}70}{\log_{10}(100) - \log_{10}(70)} = \frac{6-3}{6-\log_{10}N}$$

$\Rightarrow N = 281913.5$ rev's (or) cycles.

Stress concentration



$$k_t = \frac{\text{Max}^m \text{ stress near the discontinuity}}{\text{Nominal stress near the discontinuity}}$$

$$= \frac{\sigma_{max}}{\sigma_n} = \frac{90}{60} = 1.5$$

- Nominal stress is obtained using basic som eqns.
- Max^m stress near the discontinuity is obtained using photo elasticity technique (experimentally)
- k_t is the f^n of shape & size of discontinuity.

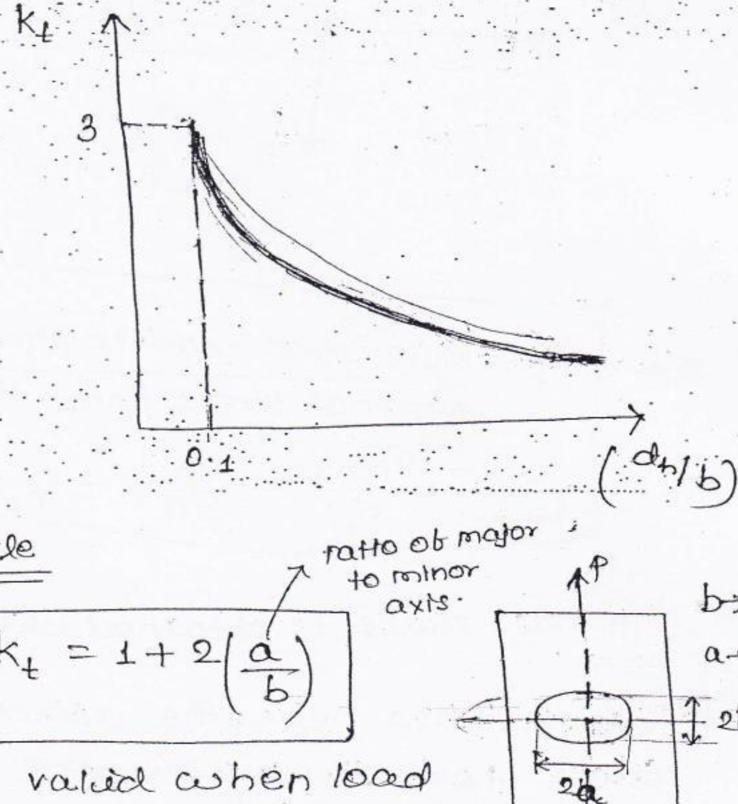
  $k_t \rightarrow$ Theoretical stress conc^n factor.
- k_t is called theoretical because material used is not considered, a model is prepared and through that model k_t is obtained.

K_f = fatigue stress concn factor.
(Give actual stress concn factor).

$$\boxed{K_t \propto F\left[(\text{shape \& size of discontinuity}) \ \& \ (\text{type of load})\right]}$$

- K_t value is independent of material.
- These discontinuity is also called stress raisers.

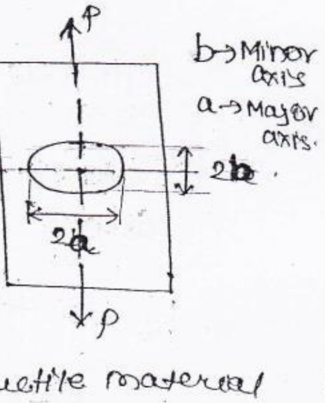
- $\frac{d_h}{b}$ = Ratio of dia of hole and width of plate.



<u>Elliptical hole</u>

ratio of major to minor axis.

$$\boxed{K_t = 1 + 2\left(\frac{a}{b}\right)}$$

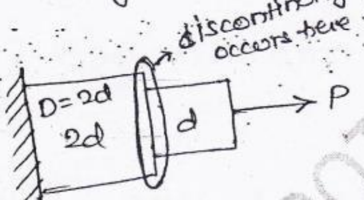


This exp. is valid when load is acting along the minor axis of hole.

** Generally, effect of stress concn in ductile material under static loading is less serious because the geometry near the discontinuity is rearranged due to <u>local yielding</u>. Hence, stress concn effect in ductile material under static loading is neglected.

- Effect of stress concn in brittle materials under static loading is more serious because they does not permit any yielding. Hence, stress concn factors should be considered in the design of brittle material under static loading.
- Effect of stress concn under fatigue loading is more serious in both ductile and brittle material. Hence, stress concn factors (K_f) should be considered in the design of a component under fatigue loading.

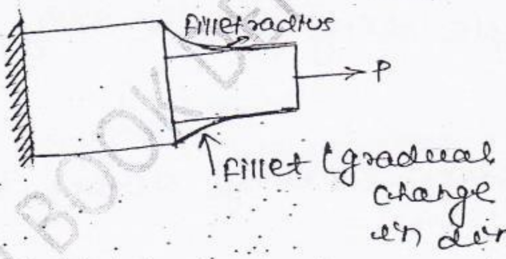
yielding is restricted only (more stress concn).



$K_t = 3$

$$\sigma_n = \frac{4P}{\pi d^2}$$

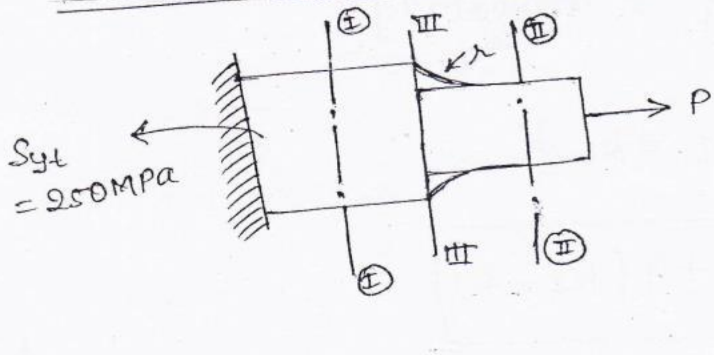
$$\sigma_{max} = K_t\left[\frac{4P}{\pi d^2}\right].$$



$K_{t_2} = 2.5$

$$\boxed{r \uparrow \Rightarrow K_t \downarrow \Rightarrow \sigma_{max} \downarrow \Rightarrow \text{chances of failures} \downarrow}$$

Assume a Mild steel bar



- yielding occur at (III) - (III).
- Total component undergo permanent deformation is called generalised yielding.

$$\sigma_{I-I} = \frac{4P}{\pi D^2}$$

$$\sigma_{II-II} = \frac{4P}{\pi d^2}$$

$$\boxed{\sigma_{max} = \sigma_{III-III} = (K_t)\left(\frac{4P}{\pi d^2}\right)}$$

local yielding occur near the discontinuity or stress concn zone.

$$P\uparrow \Rightarrow \sigma_{max} = \sigma_{III-III} \geq S_{yt}$$

$\Rightarrow$ local yielding ~~either~~ will occur near the discontinuity.

$\Rightarrow$ Geometry is rearranged $(r\uparrow) \Rightarrow (K_t \downarrow)$

$\Rightarrow = 1.5$

- If the material is brittle like cast iron, no yielding occur only fracture occur.

Fatigue failure always occur before yield point.

$K_f \rightarrow$ Fatigue (or) Actual stress concentration factor.

(or) Fatigue strength reduction factor.

$$k_f \propto [K_t \;\&\; \underset{\Rightarrow q}{\text{material}}].$$

$$k_f \propto f[q \;\&\; k_t].$$

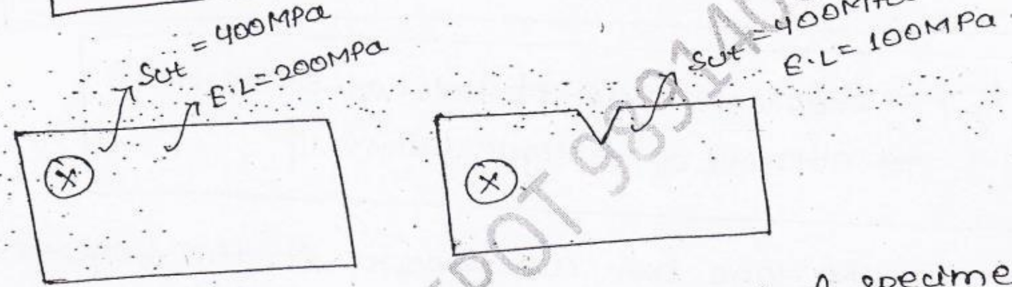
** $$\boxed{K_f = 1 + q(K_t - 1)}$$

$$\boxed{\begin{aligned} q &= \text{notch sensitivity index} \\ &= \frac{K_f - 1}{K_t - 1} \end{aligned}}$$

$q = 0 \Rightarrow K_f = 1 \Rightarrow$ Insensitive to the notches

$q = 1 \Rightarrow K_f = K_t \Rightarrow$ Too sensitive to the notches.

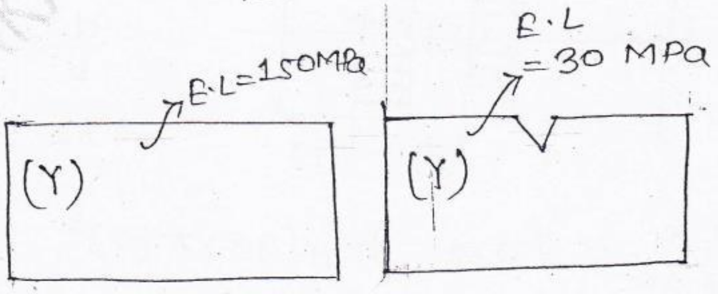
$$\boxed{0 < q < 1}$$



$$K_f = \frac{\text{Endurance limit of unnotched specimen}}{\text{E.L of notched specimen}}$$

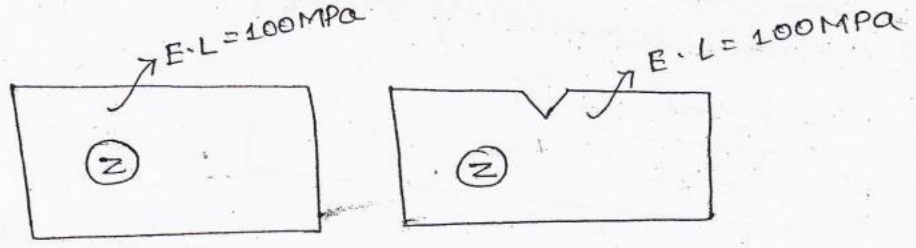
Notch is representing discontinuity.

$$K_f = \frac{200}{100} = 2$$



material is different.

$$\boxed{(K_f)_Y = 5}$$

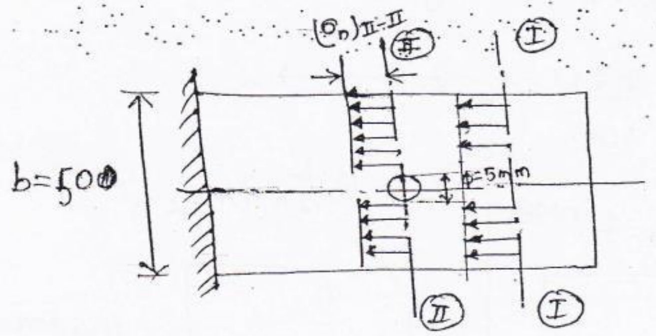


$$\left(K_f\right)_z = \frac{100}{100} \Rightarrow \text{insensitive to notch.}$$

Endurance limit of notched specimen $= \left(\frac{1}{K_f}\right)$ (E.L. un-notched specimen)

$K_f \uparrow \Rightarrow$ Endurance limit of notched specimen $\downarrow$
$\Rightarrow$ chances of fatigue failures $\uparrow$

(Q) For the long bar as shown in the figure. The stress at the section (I)-(I) is 90MPa. The stress at uniform throughout. The maxm stress at the section (II)-(II) is



$\frac{d}{b} = 0.1$

$\Rightarrow k_t = 3.$

Sol:- $\cancel{\frac{4P}{\pi d^2} = 90 \Rightarrow P = 90 \times \pi (5)^2}$

$$(\sigma_n)_{I-I} = 90\text{ MPa} = \frac{P}{bt} \Rightarrow P = 90 \times 50 \times t$$
$$\Rightarrow P = 4500t.$$

$$(\sigma_{max})_{II-II} = k_t(\sigma_n)_{II-II}$$

$$= 3\,\frac{P}{(50-5)t} = 3 \times \frac{4500t}{45t}$$

$$= 300\,\text{MPa}.$$

If the nominal stess is given at II – II,

then $\sigma_{max} = k_t(\sigma_n) = 3(90)$

$= 270\,\text{MPa}.$

Design equations used under fatigue loading [For interview imp.]

(Soderberg, Goodman & Gerber eqns)

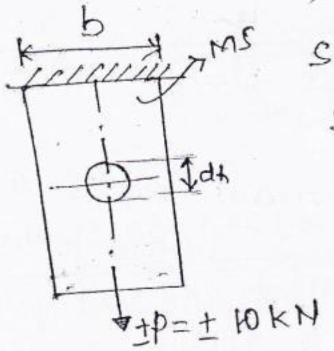
These eqn are not required under completely reversed ~~to~~ fatigue load

[Alternating fluctuating repeated fatigue load → Mean stess and variable stress both are non-zero] these eqns are valid for this case.

These eqns are must when both σ_m and σ_v are acting simultaneously (i.e, under fluctuation, repeated & alternating fatigue loads)

[optional for completely reversed load conditions (i.e, $\sigma_m = 0$; $\sigma_v = \sigma_{max}$)] [failure stess is known)

(I) case



$S_{yt} = 250\,\text{MPa}$

$S_{ut} = 400\,\text{MPa}$

det. $t = ?$

if $b = 100\,mm$

$d_h = 10\,mm.$

$$\sigma_m = 0;\ \sigma_v = \sigma_{max} = \frac{P_{max}}{(b-d_h)t} = \frac{x}{t} \quad \text{— (I)}$$

condition for safe design,

$$(\sigma_{max})_{ind} \leq \sigma_{per}$$

$$k_f \sigma_v \leq \frac{\text{failure stress}}{N}$$

$$k_f\left[\frac{x}{t}\right] \leq \frac{(\sigma_e^*)K_a K_b K_c}{N}$$

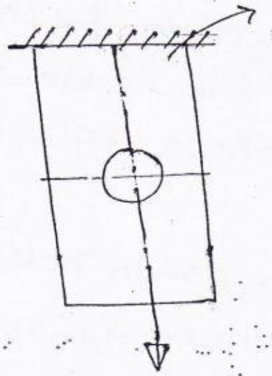
$(0.5\,S_{ut})$ 0.7

$t \geq$ ___ mm

$k_f = 2$

$N = 2$

Case II



$S_{yt} = 250\,MPa$

$S_{ut} = 40\,MPa$

$P_{max} = 2P$

$P_{min} = P$

$P_m = 1.5P$

$P_v = 0.5P$

2p top

$$\sigma_m = \frac{P_m}{(b-d_h)t} = \frac{x}{t}$$

$$\sigma_v = \frac{P_v}{d(b-d_h)t} = \frac{Y}{t}$$

Here, soderberg, Goodman eq^{n}s are must beccause failure stress is unknown.

Gerber eqn

X → Mean stress (static loading representation)
Y → variable stress (fatigue " ") [$\sigma_m = 0$]

Max^m value of σ_m is S_{yt} for ductile material & S_{ut} for brittle material under static loading.

On x-axis → $\sigma_v = 0 = \frac{\sigma_{max} - \sigma_{min}}{2}$

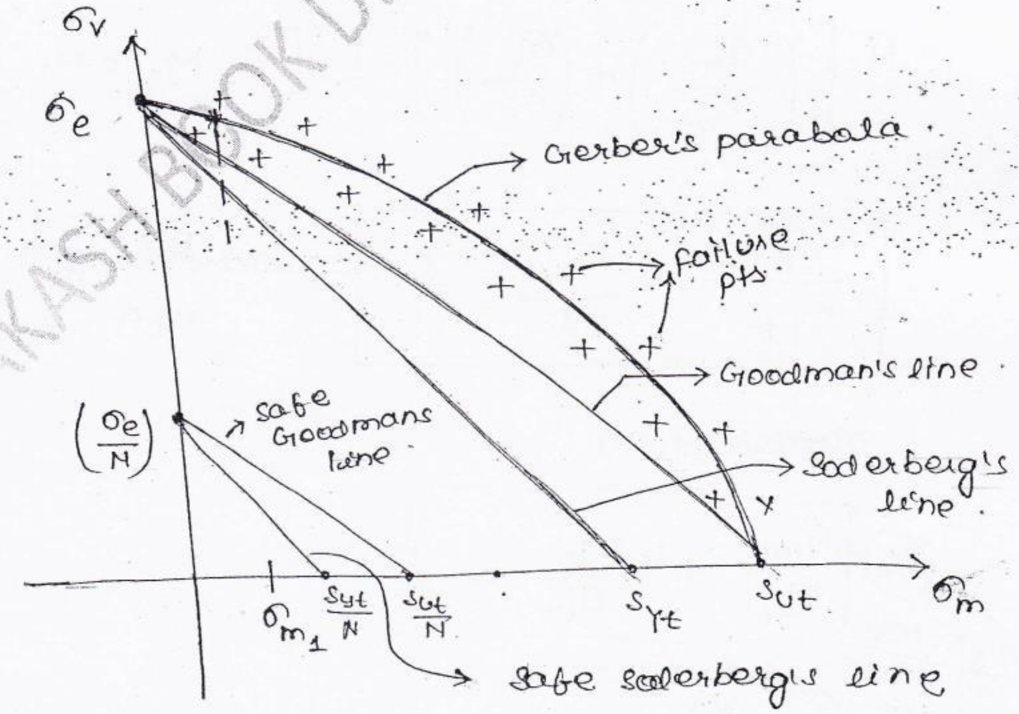
$\Rightarrow \sigma_{max} = \sigma_{min}$

On Y-axis → $\sigma_m = 0 = \frac{\sigma_{max} + \sigma_{min}}{2}$

$\Rightarrow \boxed{\sigma_{max} = -\sigma_{min}}$

[Mag. same but dir^n diff.

thus fatigue load (completely reversed) and the C.R.F.L Endurance limit is the failure stress.



Drawback of Gerbers eqn

Passing b/w the failure pts. thus not give safe design for whole component but only a part.

- uneconomic for Brittle material → soderberg's line.
- Soderberg's eqn give a always safe design.
- Design is done on the basis of permissible stress.
- These lines give failure stress, thus for getting σ_{per}, $\sigma_{failure}$ is divided by factor of safety.

$$\frac{S_{yt}}{N}, \frac{S_{ut}}{N} \rightarrow \text{permissible stresses.}$$

If FOS is diff' use this eqn.

$$\boxed{\frac{\sigma_m}{\left(\frac{\sigma_{yt}}{N_1}\right)} + \frac{\sigma_v}{\left(\frac{\sigma_e}{N_2}\right)} = 1}$$

$N_1 \rightarrow$ Factor of safety under static loading.
$N_2 \rightarrow$ FOS under fatigue loading.

$$\Rightarrow N_1\left(\frac{\sigma_m}{\sigma_{yt}}\right) + N_2\left(\frac{\sigma_v}{\sigma_e}\right) = 1$$

$$N\left[\left(\frac{\sigma_m}{\sigma_{yt}}\right) + \left(\frac{\sigma_v}{\sigma_e}\right)\right] = 1 \quad [\because N_1 = N_2].$$

$$\boxed{\frac{\sigma_m}{\sigma_{yt}} + \frac{\sigma_v}{\sigma_e} = \frac{1}{N}} \Rightarrow \text{soderberg's eqn}$$

Modification

$$\boxed{\frac{\sigma_m}{\sigma_{yt}} + k_f \frac{\sigma_v}{\sigma_e} = \frac{1}{N}}$$

k_t

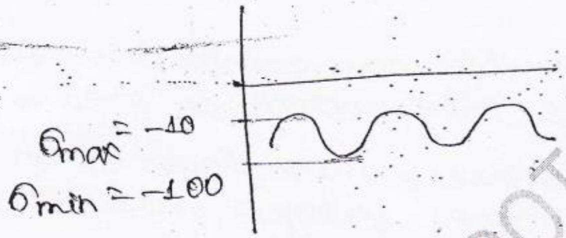
Neglected as under static loading ductile material k_t is neglected.

Goodman's eqn

$$\boxed{K_t\left(\frac{\sigma_m}{\sigma_{ut}}\right)+k_f\left(\frac{\sigma_v}{\sigma_e}\right)=\frac{1}{N}}$$

Both ~~a~~ the eqns are valid only when FOS are same.

This graph represents:- Distribution of ~~area~~ failure points when σ_m and σ_v are tensile.

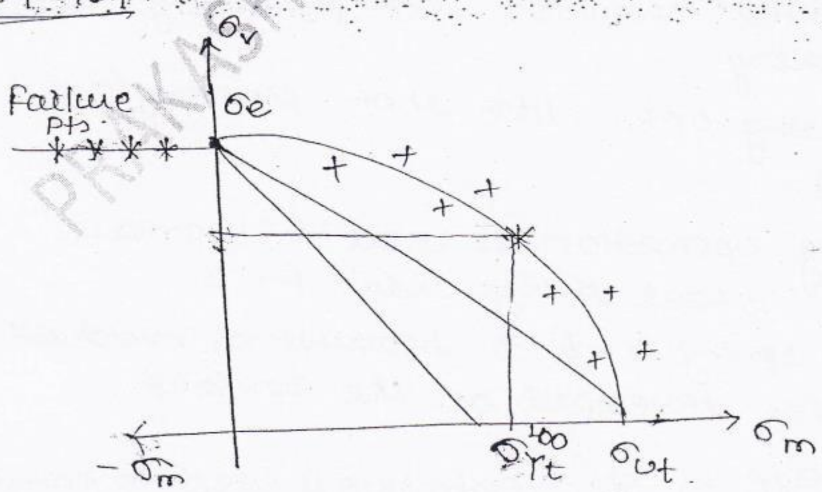


Mean stress can be -ve or +ve but variable stress is always +ve.

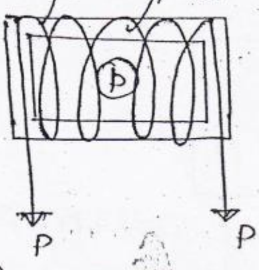
$$\sigma_m = \frac{-10-100}{2} = -55$$

$$\sigma_v = \frac{-10-(-100)}{2} = 45$$

15/01/14



σ_{hoop} σ_{comp^n} (Let $-x$ MPa)



$$\sigma_h \leq \sigma_{per}$$

$$\frac{pD}{2t} \leq \sigma_{per}$$

$$p \leq \frac{2t}{D}\sigma_{per} \rightarrow p_{max} = 10\,MPa$$

$$\sigma_h = (0_+)\,MPa \rightarrow 100\,MPa$$

$\sigma_t = -x\,MPa$.

$\sigma_h = 3x\,MPa$.

$\sigma_{TCH} = \sigma_t + \sigma_h = 2x\,MPa$.

Surface finish operation (Nitriding).
Auto frettage (for providing residual comp. stress).

From the above diagram we can conclude that fatigue failures are independent of mean comp. stresses but fatigue failure depends on mean tensile stress. Hence, fatigue failure are very rare when mean stresses are compressive in nature [i.e, fatigue failure are more likely to occur when mean stress is tensile in nature,

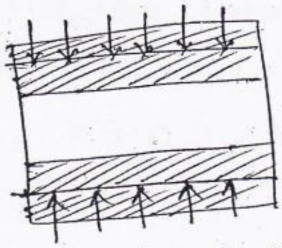
Fatigue life or F.L can be increased by following three methods.

(i) surface treatment operation like nitriding, carburising case hardening

(ii) Cold working ope. like shot pinning & burnishing.
Shot pinning operation is used to increase the life of steel springs under F.L
Life of the spring ↑ because of residual Comp. stresses developed on the surface.

(iii) <u>Auto frettage</u> → This technique is used to provide residual comp. stresses in the component.
Residual comp. stresses can be provided by following operations:-

(i) wire winding (used in thin pressure vessels)
(ii) compound cylinder (one cylinder driven inside another)
used mainly in thick pressure vessels.



compound cylinder.

(3) The peak bending stress at a critical section of a component varies b/w 100 & 300 MPa. S_{ut} = 700 MPa, S_{yt} = 500 MPa, endurance limit for reverse bending = 350 MPa. Determine FOS.

$$\frac{\sigma_m}{\sigma_{yt}} + \frac{\sigma_v}{\sigma_e} = \frac{1}{N}.$$

σ_{max} = 300 MPa, σ_{min} = 100 MPa.

$$\sigma_{mean} = \frac{\sigma_{max} + \sigma_{min}}{2} = \frac{400}{2} = 200\text{ MPa}.$$

$$\sigma_v = \frac{\sigma_{max} - \sigma_{min}}{2} = \frac{200}{2} = 100\text{ MPa}.$$

Both S_{ut} & S_{yt} are given thus the material is ductile thus Soderberg eqn is used.

$$\sigma_e = \sigma_e^* (k_a k_b k_c) \quad \left[\sigma_e^* = \tfrac{1}{2} S_{ut}\right].$$

$$= 350 (1 \times 1 \times 1)$$

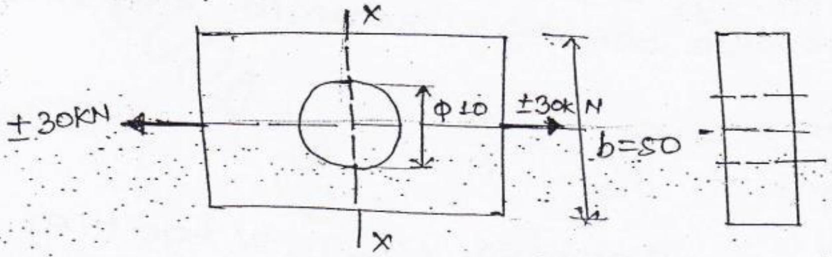
$$= 350\text{ Mpa}.$$

$$\frac{1}{N} = \frac{200}{500} + \frac{100}{350}(1)$$

$$\Rightarrow N = 1.45.$$

⑧ A plate made of steel is subjected to completely reversed axial load 30KN as shown in the fig. Determine thickness of the plate by assuming notch sensitivity index = 0.8, $K_t = 2.5$, E.L = 220 MPa, $K_a = 0.85$, $K_b = 0.897$, $K_c = 0.67$, FOS = 2.

Sol :- $P = 30KN$, $q = 0.8$ (a) 29mm
$t = ?$, $K_t = 2.5$ (b) 35mm
(c) 29.5 mm
(d) 25mm



σ_{max} exist near the discontinuity.

$\sigma_m = 0$ (Thus any eqn can be used)

$$(\sigma_{max})_{id} \leq (\sigma_t)_{per}$$

$$\boxed{k_f \sigma_v \leq \frac{\sigma_e}{N}}$$

$\boxed{k_f = 1 + q(k_t - 1) = 2.2}$

$\sigma_v = \sigma_{max} = \frac{P}{(b-d)t}$

$= \frac{30000}{(50-10)t} = \frac{750}{t}$ MPa.

$\sigma_e = \sigma_e^* k_a k_b k_c = 112.385$ MPa.

$t \geq 29.35 \Rightarrow t = 30mm$.

Result is same by using any eqn.

$$\frac{1}{N} = \frac{\sigma_m}{\sigma_{yt}} + k_f\left(\frac{\sigma_v}{\sigma_e}\right) \quad \text{(Soderberg eqn)},$$

$$\frac{1}{N} = k_f\left(\frac{\sigma_m}{\sigma_t}\right) + k_f\left(\frac{\sigma_v}{\sigma_e}\right) \quad \text{(Goodman eqn)}$$

$$\frac{1}{N} = k_f\left(\frac{\sigma_m}{\sigma_{yt}}\right)^2 N + k_f\left(\frac{\sigma_v}{\sigma_e}\right) \quad \text{[Gerber eqn]}$$

$$\sigma_m = 0 \Rightarrow \frac{1}{N} = k_f \frac{\sigma_v}{\sigma_e}$$

$$\Rightarrow \frac{\sigma_e}{N} = k_f \sigma_v.$$

Imp
- for both ductile and brittle materials soderberg, Goodman and Gerber give same result under completely reversed loading conditions.
- under completely reversed loading conditions, design of a component is similar to the design under static loading. Hence, for completely reversed loading conditions Soderberg, Gerber and Goodman eqn are optional.

W.B (Q7) $D = 200mm, t = 1mm$

$p = 4$ to 8 MPa

$S_{yt} = 600MPa, S_{ut} = 800MPa, \sigma_e^* = 400MPa, N = ?$

$$\sigma_h = \frac{pD}{4t}$$

$$p_m = 6MPa = \frac{8+4}{2} = 6$$

$$p_v = \frac{8-4}{2} = 2$$

$$(\sigma_m)_h = \frac{p_m D}{4t} = \frac{6 \times 200}{4 \times 1} = 300MPa.$$

$$(\sigma_v)_h = \frac{p_v D}{4t} = \frac{2 \times 200}{4 \times 1} = 100MPa.$$

$$\frac{1}{N} = \underbrace{K_t}_{1}\left(\frac{\sigma_m}{\sigma_{ut}}\right) + \underbrace{K_f}_{1}\left(\frac{\sigma_v}{\sigma_e}\right)$$

$$= \left(\frac{300}{600} + \frac{100}{400}\right) = \cancel{0.75}\ 1.6$$

G-13 (Q) A bar is subjected to fluctuating tensile load from 20kN to 100kN. The material has yield strength of ~~200~~ 240MPa and E.L in reverse bending is 60 MPa. According to soderberg principle the area of C/s in mm² of the bar for a FOS = 2.

Sol $P_{max} = 100kN, P_{min} = 20kN$

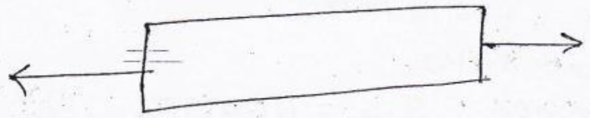
$S_{yt} = 240MPa, E.L = 60MPa.$

$$\cancel{K_f \sigma_v \le \frac{\sigma_e}{N}} \Rightarrow \cancel{\frac{80}{A} \times 2 \le \frac{60}{2}}$$

$$\sigma_v = \frac{\frac{P_{max}}{A} - \frac{P_{min}}{A}}{2} \Rightarrow \cancel{40}\ \frac{160}{A} = \frac{\cancel{60}}{2}\,30$$

$$\Rightarrow A =$$

Φ.



20 to 100 kN

20 kN to 100 kN

$$P_{mean} = \frac{100+20}{2} = 60\,kN.$$

$$P_v = \frac{100-20}{2} = 40\,kN.$$

$$\sigma_m = \frac{P_m}{A} = \frac{60000}{A}\ MPa.$$

$$\sigma_v = \frac{P_v}{A} = \frac{40000}{A}\ MPa.$$

$$\frac{1}{N} = \frac{\sigma_m}{\sigma_{yt}} + K_f\left(\frac{\sigma_v}{\sigma_e}\right)$$

$$\sigma_e = \sigma_e^* K_a K_b K_c$$

$$= 160 \times 1 \times 1 \times 0.7 = 112\ MPa.$$

$$\frac{1}{2} = \frac{60000}{A(240)} + (1)\left(\frac{40000}{A \times 112}\right)$$

~~A = 121404.76 mm²~~

$$A = 1214.28.$$

(a) 400 mm² (b) 600 mm²

(c) 750 mm² (d) 1000 mm² ✓

(Q) A 50mm dia. shaft made from carbon steel is subjected to a torque which fluctuates b/w 2KN-m to −8 KNm. FOS a/c to soderberg criterion if S_{ys} = 220MPa & E.L = 150MPa

If one tensile & another compressive then it is Alternating fatigue

Sol → D = 50 mm

(when Bending moment acting alone)

$$\frac{1}{N} = \left(\frac{\sigma_m}{\sigma_{yt}}\right) + k_f\left(\frac{\sigma_v}{\sigma_e}\right)$$

$$\frac{1}{N} = k_t\left(\frac{\sigma_m}{\sigma_{ut}}\right) + k_f\left(\frac{\sigma_v}{\sigma_e}\right)$$

valid only for one type of loading either it may be axial load, bending moment or twisting moment

For twisting moment

(when twisting moment acting alone)

$$\frac{1}{N} = \frac{\tau_m}{\tau_{ys} \text{ (or) } S_{ys}} + k_f \frac{\tau_v}{\tau_e}$$

S_{ys} → A/c MSST → $\frac{S_{yt}}{2}$

A/c MDET → $\frac{S_{yt}}{\sqrt{3}}$

$$\tau_e = \sigma_e^* \, k_a \, k_b \, k_c$$

→ for T.M $k_c = 0.6$

T_{max} = 2KN-m = 2×10^6 N mm

T_{min} = -0.8×10^6 N-mm.

$$\tau_m = \frac{16 T_m}{\pi d^3} = \frac{16 \times 10^6}{\pi (50)^3} = \cancel{40.74}\text{ MPa} = 81.48 \text{ MPa}.$$

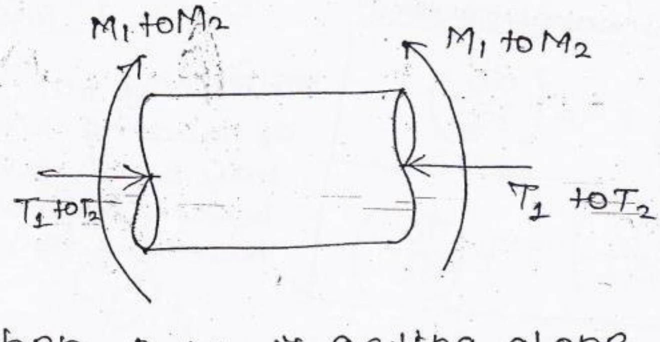
$$\tau_v = \frac{16 T_v}{\pi d^3} = \frac{16 \times -0.8 \times 10^6}{\pi (50)^3}$$

$= -32.59$ MPa.

$$\tau_e = \sigma_e^* k_a k_b k_c = 150 \times 1 \times 1 \times 10^6 \, 0.6,$$
$$= 90 \, MPa$$

$$N = 1.35$$

(9)



When B.M is acting alone

$$M_m = \_\_\_\_$$
$$M_v = \_\_\_\_$$

$$\sigma_m = \frac{32 M_m}{\pi d^3} = \frac{X}{d^3} \, MPa.$$

$$\sigma_v = \frac{32 M_v}{\pi d^3} = \frac{Y}{d^3} \, MPa.$$

$$\frac{1}{N} = \frac{\sigma_m}{\sigma_{yt}} + k_f \frac{\sigma_v}{\sigma_e}$$

$$\frac{\sigma_{yt}}{N} = \sigma_m + k_f \frac{\sigma_v \sigma_{yt}}{\sigma_e}$$

$$(\sigma_{ea})_{bend} = \sigma_m + k_f \frac{\sigma_v \sigma_{yt}}{\sigma_e}$$

$$= \frac{Z}{d^3} \, MPa \quad \text{--- (f)}.$$

when T.M acting alone

$$T_m = \_\_\_\_$$
$$T_v = \_\_\_\_, \quad \tau_m = \frac{16 T_m}{\pi d^3} = \frac{x'}{d^3}.$$

$$\tau_v = \frac{16}{\pi d^3} = \frac{Y'}{d^3}.$$

$$\frac{1}{N} = \frac{\tau_m}{\tau_{ys}} + k_f \frac{\tau_v}{\tau_e}$$

$$\frac{\tau_{ys}}{N} = \tau_m + k_f \frac{\tau_v \tau_{ys}}{\tau_e}$$

$$\boxed{\tau_{eq} = \tau_m + k_f \frac{\tau_v \tau_{ys}}{\tau_e} = \frac{Z'}{d^3} \text{ MPa}} \quad (\text{II})$$

MSST,

$$\tau_{per} = \frac{S_{ys}}{N} = \frac{1}{2}\sqrt{(\sigma_{eq})^2 + 4(\tau_{eq})^2}$$

$$\boxed{d = ?}$$

MDET,

$$\tau_{per} = \frac{S_{ys}}{N} = \frac{1}{\sqrt{3}}\sqrt{(\sigma_{eq})^2 + 3(\tau_{eq})^2}$$

$$\Rightarrow \boxed{d = ?}$$

For objective questions

$$\boxed{\frac{\sigma_{yt}}{N} = \sigma_m + k_f \frac{\sigma_v \sigma_{yt}}{\sigma_e}}$$

(Q) A shaft is subjected to normal stress which is fluctuating b/w -130 MPa to $+130$ MPa. [$\sigma_m = 0$, comp. rever.] and Torsional stress which is fluctuating b/w 16 MPa to 54 MPa. Determine FOS using Distortion energy theory by using soderberg's relation. Assume $K_t = 1.85$ and $S_{ut} = 800$ MPa, $S_{yt} = 600$ MPa, $q = 0.95$.

Sol:- $\sigma_m = 0$, $\sigma_v = \sigma_{max} = 130$ MPa

$$\sigma_{eq} = \sigma_m + k_f \frac{\sigma_v \sigma_{yt}}{\sigma_e}$$

$$\sigma_{eq} = 0 + \frac{k_f (130) 600}{0.5 \times 800 \times 1 \times 1 \times 1} = 352.46 \text{ MPa}$$

$$\tau_m = \frac{16 + 57}{2} = 36.5 \text{ MPa}$$

$$\tau_v = \frac{57 - 16}{2} = 20.5 \text{ MPa}$$

$$\tau_{eq} = \tau_m + \frac{k_f \, \tau_v \, \tau_{ys}}{\tau_e} \Rightarrow S_{yt}/\sqrt{3} =$$

$$\tau_{ys} = \frac{S_{yt}}{\sqrt{3}} = \frac{600}{\sqrt{3}} = 346.4 \text{ MPa}$$

$$\Rightarrow \tau_e = 0.5 \times 800 \times 1 \times 1 \times 0.6$$
$$= 240 \text{ MPa}$$

$$\frac{S_{ys}}{N} = \frac{1}{\sqrt{3}} \sqrt{(\sigma_{eq})^2 + 3(\tau_{eq})^2}$$

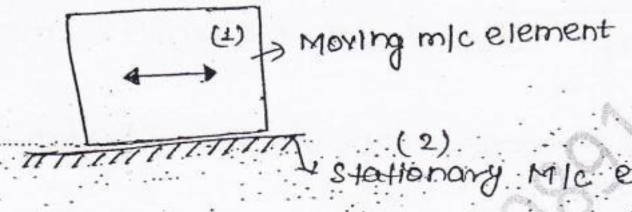
$$N = \frac{346.4 \times \sqrt{3}}{\sqrt{(352.46)^2 + 3(240)^2}} = 1.1$$

$$k_f = 1 + q(k_t - 1)$$
$$= 1 + 0.95(0.85) = 1.8075$$

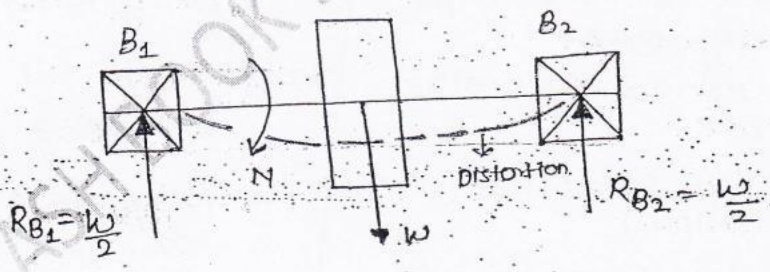
Bearings

w.r.t to lathe carriage, lathe bed is treated as bearing. The stationary object w.r.t to some moving body is called bearing.

Whenever relative motion occurs b/w two m/c elements, the machine element which is stationary and supporting the moving m/c element is known as bearing.



Bearing is a m/c element whose fn is to support a rotating m/c element (i.e shaft) and to guide or confines its motion while preventing its motion in the dirn of applied load.



Drawback of bearing → power loss, wearing (because of relative motion).

A bearing is a good bearing when it performs its function without power loss. Power loss and wearing can be reduced by using lubricants.

fn of lubricant :-

carry away heat, power loss ↓, wearing ↓, corrosion ↓.

lubricant :- viscous fluid.

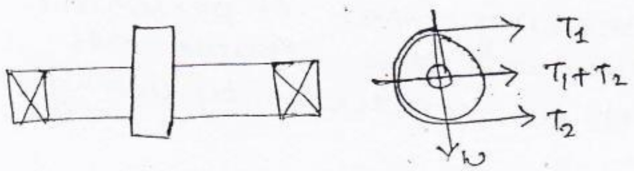
- Because of the relative motion b/w shaft and bearing surfaces always some amount of power loss occurs in overcoming the frictional resistance and wear of the surfaces also occurs due to metal to metal contact.
- A Bearing is said to be a good bearing which performs its given functionality (i.e, supporting the shaft) with minimum power loss and wear. This is possible by providing lubrication b/w shaft and bearing surfaces.

Functions of Bearings

- To support shaft and to hold in its correct position.
- To ensure free rotation of the shaft with minm friction.
- loads acting on the shaft and to transmit them to the foundation or frame of the machine.

SHAFT

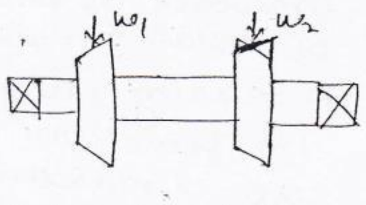
- Both B.M & T.M
  ~~only B.M~~
- Designed by using T.OF
- only circular (may be solid or hollow)
- Shafts are used to transmit the power.



Subjected to horizontal & vertical T.S.L & T.M

AXLE

- only B.M
- Designed ~~only~~ by only Bending eqn
- May be circular or non "
- Axle is a supporting member.



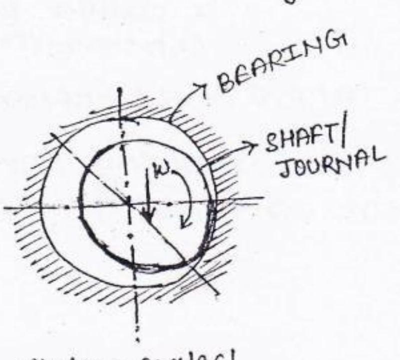
Subjected to T.S.L only.

Automobile Axle is circular and circular c/s must be hollow in case of Axle.

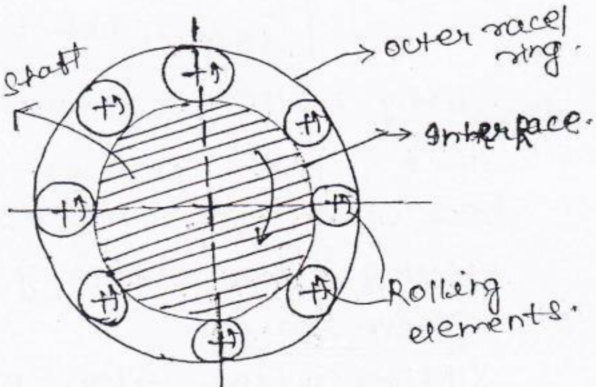
BEARINGS

- SLIDING (S.C.B) CONTACT BEARING
  - Nature of sliding action
    - SLEEVE BEARING
      - dirn of load
        - THRUST BEARINGS
        - RADIAL (OR) JOURNAL BERINGS
          - configuration of Bushes
            - SOLID BUSH Bearing
            - Split Bush Bearing
      - LUBRICATION
        - THICK FILM LUBRICATION
          - HYDRODYNAMIC LUB.
          - HYDROSTATIC LUB. (Ext. pressurized Lub.)
        - THIN FILM LUBRICATION (Boundary Lub.)
        - ZERO FILM LUB. (Self Lub)
    - SLIPPER (OR) GUIDE BEARING
- ROLLING CONTACT BEARINGS (Anti-friction Bearing)
  - SHAPE OF ROLLING ELEMENTS
    - BALL BEARINGS (B.B)
      - Deep Groove Ball Bearing (D.G.B.B)
      - Self Aligning B.B.
      - Angular Contact Bearing.
      - THRUST B.B
    - ROLLER BEARING (RB)
      - Cylindrical R.B
      - Tapered R.B
      - Spherical R.B
      - Needle R.B

- Shaft inside the bearing is called journal



Sliding contact Bearing



Anti friction Bearing

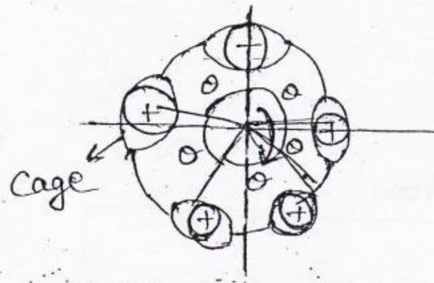
Journal

A portion of shaft lies in the bearing.

Journal dia = Shaft dia	
Length of the Journal = Length of the Bearing	

- Outer race rotate and inner race is stationary in case of hollow sphere shaft.

Working of cage



- To prevent clustering of rolling of roller element.
- Maintain const. relative angular position of the rolling elements b/w adjacent rolling elements.
- Holds the rolling element and avoid contact b/w the adjacent roller.
- To seperate the adjacent rolling element.

Cage is used to hold the rolling elements together and spaces them evenly around the periphery of shaft.

Cage is also called seperator or Retainer.

Cage is absent in Needle roller bearing.

- Anti-friction bearing is a misnomer (where there is contact, friction exist).

Power loss eqn (occurs due to frictional force.)

$$P_{loss} = T_f \omega$$
$$= F_f r \omega$$
$$\boxed{P_{loss} = \mu W V}$$

- In clutches friction force are helpful but here it is harmful. In clutches it is called power absorbtion.

Why rolling contact is called Anti friction :-
Since, the power loss in rolling contact bearing is very less in comparision to sliding contact bearing.

Sleeve bearing
Sliding action takes place along arc of circle,

Slipper bearing sliding action takes place along

a straight line.
In shaft always sleeve bearing is used.

THRUST BEARING

load is acting along the axis of the shaft (generally axial compressive load)

Radial or journal bearing

used to support a shaft subjected where load is acting perpendicular to shaft axis.

Thin film lubrication

where partial metal to metal contact takes place. used in m/c tool.

Thick film lubrication

No metal to metal contact (thus this is better) used in power transmission.

Self lubrication

used in food processing industries; no external lubrication. Bearings have ~~atta~~ itself self lubricating characters. eg:- Graphite. chemically inert even at elevated temp.

Hydrodynamic

lubricant enters at atm. pressure, initially pressure exerted by the lubricant is less. suitable only at high speed application.

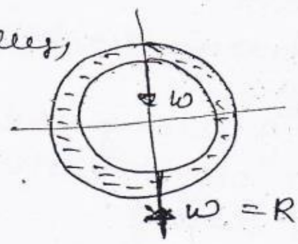
wedging or convergent action

enters bearing at atm press. Moving of lubricant from wider to narrow region is called wedging or convergent action. Pressure increases due to this and thus hydrodynamic lubn is not suitable for low speed or medium speed applications.

Drawback of Hydrodynamic

Not suitable when heavy load is present initially.

- load very high initially, power loss is very high.
  In stone crushers same phenomenon is used.



Starting torque is minm

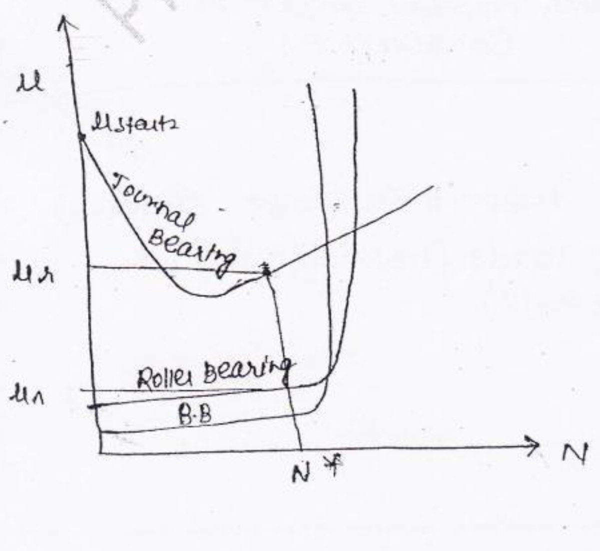
Hydrodynamic	Hydrostatic Lub.
• High starting torque	• less starting torque
• No - eccentricity used in application when lighter loads initially. used in hydrodynamic i.e engine	• No Eccentricity present. Higher loads initially
(1) Lubricant is supplied at atmospheric pressure	(1) Lubricant is supplied at higher pressure.
(2) pressure of the lubricant increases due to dynamic or wedging action.	(2) Pressure of the lubricant increases due to external device like a pump.
(3) Also known as self acting lubrication.	(3) Also known as extenally pressurized lubrication.
(4) Metal to metal contact b/w shaft and bearing surfaces can be avoided at the high speed.	(4) There is no possibility of metal to metal contact b/w shaft and bearing surface (i.e, even ath the stationary condition.
(5) High Starting torque is required.	(5) Starting torque is minm.
(6) shaft rotates eccentrically w.r.t to the bearing.	(6) shaft rotates concentrically w.r.t bearing.

(7) cost of lubrication is less.	(7) High initial and maintenance cost.
(8) It is used in the applications where shafts are subjected to lighter load at initial condion. Eg:- I.C engine crank shaft & most of the industrial machinery.	(8) used in a application where shafts are subjected to heavier load at the stationary condition. Eg:- Ball ~~or~~ mills used in power plants & cement plant. centrifuses and vertical turbogeneration.

Plumber block is used to support a very lengthy shaft which requires support at intermediate location.

- load carrying capacity of Roller bearing is more than ball bearing (only pt. load).
- Roller bearing (cylendrical to this surface contact or line contact).

Journal Bearing	Rolling Bearing
• provide continuous service	• used in ~~intermediate~~ intermittent service condition.
• starting torque is more. (for high speed). used in I.C engine crank shaft.	• suitable for low and medium speed. used in Automobile, m/c tools, Household app. power loss is less. Drawback Costlier, manufacturing is difficult. suitable for ~~par~~ radial & thrust loads both.



$F_r \rightarrow$ radial load

$F_a \rightarrow$ Axial load

Deep Groove B.B

✓ High F_r, Low F_a → Deep Groove B.B $\left[\frac{F_r}{F_a} > 1\right)$

Angular

✓ low F_r, High F_a → $\left[\frac{F_r}{F_a} < 1\right]$.

Self Alignment

whenever misalignment is likely to occur b/w balls and shafts self alignment B.B is used.

THRUST B.B

only thrust load present here.

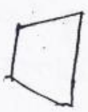
Cylindrical R.B

plane cylinder

$L/D < 1$ (only F_r) [only radial load]

Tapered R.B

$L/D < 1$ $\left(\frac{F_r}{F_a} > 1\right)$



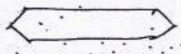
Spherical R.B

$\left(\frac{L}{D} < 1\right)$

Self aligning property.

Needle R.B

- Length to dia ratio more $\left(\frac{L}{D} > 1\right)$



- No cage.
- ~~whenever roller speed is considered.~~
- oscillating motion.
- where radial space is constraint.

16/01/14

Thrust Bearing

→ Thrust bearing are used to support a shaft subjected to an axial compressive loads (i.e, loads which are acting along the shaft axis).

THRUST BEARING

- PIVOT BEARINGS
  - Flat Pivot Bearing (foot step Bearing)
  - Conical Pivot Bearing
- Collar Bearings
  - Flat collar Bearings
    - Single Collar Bearings
    - conical Collar Bearing.
  - Conical Collar Bearings

Pivot Bearings → support vertical shaft subjected to axial comp load.

To obtain design eqns for pivot bearings subjected to $R_o = R$, $R_i = 0$ in collar bearings eqns.

Collar Bearings :-

→ Collar bearings are used to support a horizontal shaft subjected to thrust loads.

R_i = shaft radius.

R_o = ?

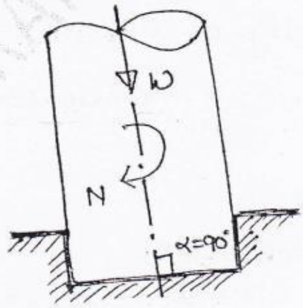


Fig. Flat pivot bearing (foot step bearing)

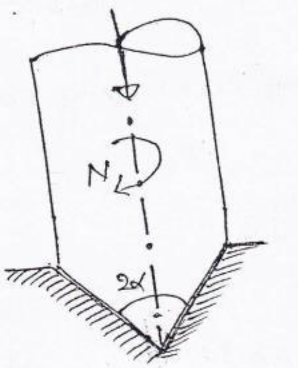
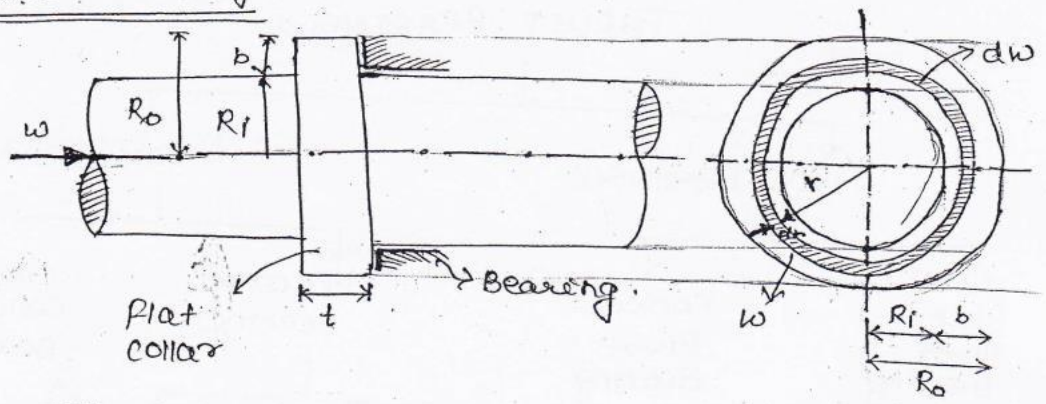


Fig. conical pivot bearing

2α = cone angle.

α = semi cone angle.

Single collar Bearing



Drawback of single collar bearing

- If the load increase, width of collar ↑, more radial space is required but area is constraint (limited space). Four wheeler vehicle
- To overcome this drawback multicollar bearing is used. Eg:- Two wheeler vechcle. ~~as~~ Here space is limited thus multicollar bearing is used. Here load is distributed and there is no effect on the radial space or on width of collar.

Single collar bearing

~~width~~ $w\uparrow \Rightarrow b\uparrow \Rightarrow R_o\uparrow \Rightarrow$ more radial space is required.

whenever radial space is constraint,

$$n\uparrow \Rightarrow W_{each}\downarrow \Rightarrow b\downarrow \Rightarrow R_o\downarrow.$$

$w \rightarrow$ load~~ing~~ carrying capacity of collar.

$dw \rightarrow$ " " " " small elemental ring.

$dA = 2\pi r\,dr$

$dw = p \times 2\pi r\,dr$

$dF_f = \mu\,dw$

$dF_f = \mu p\, 2\pi r\,dr$

$p \rightarrow$ pressure.

$dA \rightarrow$ elemental area,

$dF_f \rightarrow$ elemental frictional force.

$$dT_F = dF_r \times r = \mu p \, 2\pi r^2 dr \quad \text{— (II)}$$

$\downarrow$ elemental torque

Two theories are considered

(i) uniform pressure theory

$\boxed{p = \text{const.}}$

(ii) uniform wear theory

$\boxed{pr = \text{const}}$ i.e. $p \propto \frac{1}{r}$

Eg: [variation of pressure]

$L_1 = L$, $p_1 = p$

A ———— B $\quad \omega_1 = \omega$

$L_2 = 2L$, $p_2 = p/2$

C ———————— D $\quad \omega_2 = \omega_1$

$$p_2 L_2 = \frac{p}{2} \times 2L \qquad \boxed{p \propto \frac{1}{L}}$$

$$p_2 L_2 = p \times L$$

$$\Rightarrow p_2 L_2 = p_1 L_1 \Rightarrow pL = \text{const.}$$

uniform pressure theory (UPT)

$$W = \int_{R_i}^{R_o} dW = \int_{R_i}^{R_o} p \times 2\pi r \times dr$$

$$W = 2\pi p \int_{R_i}^{R_o} r \, dr$$

$$= \pi p \left[R_o^2 - R_i^2 \right]$$

$$W = \pi p \left[R_o^2 - R_i^2 \right]$$

$$\boxed{p_{UPT} = \frac{W}{\pi(R_o^2 - R_i^2)} \leq p_{per}} \quad \text{— (A)}$$

$$dT_f = \mu \times \frac{W}{\pi(R_o^2 - R_i^2)} \times 2\pi r^2 dr$$

$$T_f = \frac{2\mu W}{(R_o^2 - R_i^2)} \int_{R_i}^{R_o} r^2 dr$$

$$\boxed{T_f = \frac{2}{3}\mu W\left(\frac{R_o^3 - R_i^3}{R_o^2 - R_i^2}\right) = \mu W\left[\frac{2}{3}\left(\frac{R_o^3 - R_i^3}{R_o^2 - R_i^2}\right)\right]} \quad \text{— (B)}$$

Imp

$$\boxed{\frac{2}{3}\left(\frac{R_o^3 - R_i^3}{R_o^2 - R_i^2}\right) \longrightarrow \text{Effective radius } (R_{eff})_{UPT}}$$

$$\boxed{(T_f)_{UPT} = (\mu W)(R_{eff})}$$

Uniform wear theory

$$\boxed{pr = \text{const}}$$

$$W = \int_{R_i}^{R_o} dW = \int_{R_i}^{R_o} p \times 2\pi r\, dr$$

$$W = 2\pi pr \int_{R_i}^{R_o} dr = 2\pi pr\,[R_o - R_i].$$

$$\boxed{p_{UWT} = \frac{W}{2\pi r(R_o - R_i)}} \quad \text{— (C)}$$

Max^m pressure → Inner radius considered.
Min^m " → outer " "
Average " → Mean of max^m & min. radius.

If,

$r = R_i \Rightarrow p = p_{max}$

$r = R_o \Rightarrow p = p_{min}$.

$r = R_m \Rightarrow p = p_{avg}$.

$$\boxed{(p_{max})_{UWT} = \frac{W}{2\pi R_i(R_o - R_i)}}$$

$$dT_f = \mu \times \frac{W}{2\pi r(R_o - R_i)} \times 2\pi r^2 dr$$

$$dT_f = \frac{\mu W r}{(R_o - R_i)} dr$$

$$T_f = \int_{R_i}^{R_o} \frac{\mu W}{(R_o - R_i)} r\,dr$$

$$(T_f)_{UWT} = \left[\frac{\mu W}{R_o - R_i}\right]\left(\frac{R_o^2 - R_i^2}{2}\right)$$

$$\boxed{(T_f)_{UWT} = \mu W\left(\frac{R_o + R_i}{2}\right)} \quad \text{—— (D)}$$

$\boxed{\frac{R_o + R_i}{2} \Rightarrow R_{effective}}$ as per uniform wear theory.

SUMMARY

$$\boxed{p_{UPT} = \frac{W}{(R_o^2 - R_i^2)\pi}}$$

$$\boxed{p_{UWT} = \frac{W}{2\pi r(R_o - R_i)} \;;\; p_{max} = \frac{W}{2\pi R_i(R_o - R_i)}}$$

$$\boxed{(R_{eff})_{UPT} = \frac{2}{3}\left[\frac{R_o^3 - R_i^3}{R_o^2 - R_i^2}\right]}$$

$$\boxed{(R_{eff})_{UWT} = \left(\frac{R_o + R_i}{2}\right) = \text{Mean radius}}$$

$$\boxed{T_f = \mu w (R_{eff})}$$

$$\boxed{P_{Loss} = T_f \omega}$$

T_F EQ^n^s for flat Pivot Bearings

$(R_i = 0 \; ; \; R_o = R)$

$$(R_{eff})_{UPT} = \frac{2}{3}[R]$$

$$(R_{eff})_{UWT} = \frac{R}{2}$$

$$(T_F)_{UPT} = \mu w \left(\frac{2}{3}\right) R$$

$$(T_F)_{UWT} = \mu w \left(\frac{R}{2}\right)$$

$$\boxed{\frac{(T_F)_{UPT}}{(T_F)_{UWT}} = \frac{4}{3} = 1.33}$$

- If unless otherwise not mentioned then we use UPT.
- But in clutches UWT is used.
- We can conclude that frictional torque or power loss as per UPT is more than the power loss as per uniform wear theory.
  Hence, for the safe design of bearings if unless or otherwise mentioned it is better to assume uniform pressure theory because power loss

occurs in bearings in overcoming the frictional resistance.

- For the safe design of clutches (i.e, old (or) worn out clutches) it is better to use uniform wear theory because pressure is non-uniformly distributed when clutch surfaces comes into functionality and clutches are used to transmit power by utilising frictional forces.
- For the safe design of new clutches it is better to use uniform pressure theory because the pressure is uniformly distributed when the clutch surfaces are new in condition.

Thickness of collar

For safe design

$$(\tau_{max})_{ind} \leq \tau_{per}$$

$$\frac{W}{\pi D_i t} \leq \tau_{per}$$

$$\boxed{t \geq \_\_\_ \text{ mm}}$$

Multi collar bearing

$$\boxed{n = \frac{W}{W_{each\ collar}}}$$

$$n = ?$$

$$(T_f)_{\substack{Multi\\ collar\\ Bearing(M.C.B)}} = n\left[\mu W_{each} \times \frac{2}{3}\left(\frac{R_o^3 - R_i^3}{R_o^2 - R_i^2}\right)\right]$$

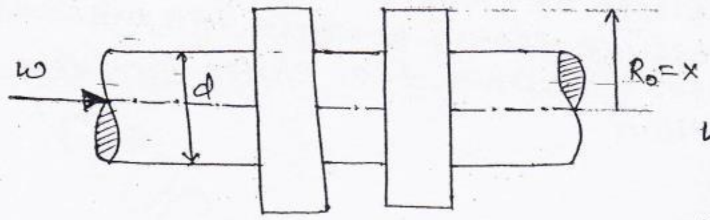
$$\boxed{(T_f)_{M.C.B} = \mu W\left[\frac{2}{3}\left(\frac{R_o^3 - R_i^3}{R_o^2 - R_i^2}\right)\right]}$$

Conclusion

Frictional torque (T_f) are independent of no. of collars.

imp

$$\boxed{n\uparrow \Rightarrow W_{each}\downarrow \Rightarrow p\downarrow}$$

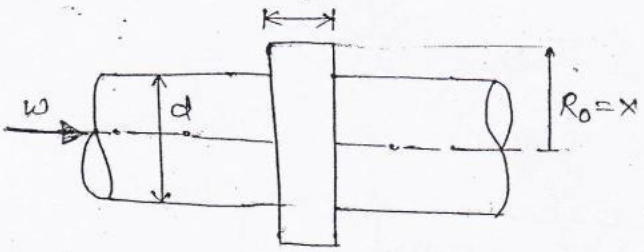


$W_{each} = W/2$

$p_{each} = p/2$.

$(T_f)_{MCB} = T$

Multi- collar bearing



$W_{each} = W$

$p_{each} = p$

$(T_f)_{S.C.B} = T$

Single collar bearing

In multicollar bearing

$(T_F)_{M.C.B}$ is used firstly and then (W)each collar eq^n.

Steps to be used in numericals

(1) $P_{LOSS} = T_f\,\omega$

$T_f =$ ______ N-m.

(2) $T_f = \mu W\left[\frac{2}{3}\left(\frac{R_o^3 - R_i^3}{R_o^2 - R_i^2}\right)\right]$.

$R_o =$ __________ mm.

(3) $W_{collar} = p_{per} \times \pi(R_o^2 - R_i^2)$

$=$ ______ N.

(4) $n = \frac{W}{W_{collar}}$

conical collar bearing (C.C.B)

(1) $p_{UPT} = \frac{W}{\pi(R_o^2 - R_i^2)} = ?$

S.C.B → Single collar bearing

C.C.B → conical collar bearing.

(2) $(T_f)_{C.C.B} = \frac{1}{\sin\alpha}[T_f]_{S.C.B}$

$\alpha \uparrow \Rightarrow \sin\alpha \uparrow \Rightarrow (T_f) \downarrow \Rightarrow P_{loss} \downarrow$ (Bearings)

$\alpha \downarrow \Rightarrow \sin\alpha \downarrow \Rightarrow T_f \uparrow \Rightarrow$ (Power Transmission capacity of conical collar)$\uparrow$

C.C.B ;	$2\alpha = 120°$ to $160°$
Cone clutches ;	$2\alpha = 25°$ to $30°$

- Pressure induced or load carrying capacity is independent of semi-cone angle but frictional torque is inversely proportional to $\sin\alpha$.

(8) which of the following statement are valid for a multi-collar thrust bearing subjected to an axial load of W?
(1) Frictional load is independent of no. of collar.
(2) Intensity of pressure is affected by no of collars.
(3) Coefficient of friction of bearing surface is affected by no. of collar.

1 & 3   2 & 3   (c) 1 & 2   (d) 1, 2, 3

(8) when the intensity of pressure is uniform in flat pivot bearing then the friction force is assumed to act at :-

(a) r (b) $\frac{r}{3}$ (c) $\frac{2r}{3}$ (UPT)   (d) $\frac{r}{2}$ (UWT)

(9) A multicollar thrust bearing has inner and outer dia. of 300mm and 400mm resp. If permissible pressure is 7KN/mm² and load acting on the shaft 1750 MN - Determine :-
(i) No. of collars required for this bearing.
(ii) Pressure developed at inner outer & at mean radius of the collar.

Sol $R_i = 150mm$, $R_o = 200mm$.

$P_{per} = 7KN$

(i) $$W_{each} = p_{per} \times \pi [R_o^2 - R_i^2].$$

$$= 70000 \times \pi [200^2 - 150^2].$$

$$= 384.84 \times 10^6 N.$$

$$= 384.84 MN.$$

(ii) $$n = \frac{W \rightarrow 1750MN}{W_{collar}} = 4.5 \simeq 5.$$

(iii) $$(p)_{x=R_i} = (p)_{x=R_o} = (p)_{x=R_m}$$

$$\left[W = p \times \pi (R_o^2 - R_i^2)\right] \quad = \frac{W}{n \times \pi (R_o^2 - R_i^2)} = \frac{1750 \times 10^6}{5 \times \pi [200^2 - 150^2]}$$

$$= 6366.19 N/mm^2.$$

$$= 6.37 KN/mm^2.$$

(9) Repeat the above question if the pressure is non-uniformly distributed on the collar.

$$W_{each} = p_{per} \times 2\pi R_i (R_o - R_i)$$

$$p_{UWT} = \frac{W}{2\pi r (p_o - p_i)} \Rightarrow (p_{max})_{ind} \leq p_{per}$$

$$\frac{W}{2\pi R_i (R_o - R_i)} \leq p_{per} \Rightarrow W_{each} \leq \underbrace{p_{per} \times 2\pi R_i (R_o - R_i)}_{W_{each}}$$

$$\therefore W_{each} = 7000 \times 2\pi(150)(200-150)$$
$$= 329.7\ MN.$$

(2) $$n = \frac{W}{W_{collar}} = 5.3 \simeq 6.$$

(3) $$(p)_{x=R_i} = \frac{W_{each}}{2\pi R_i(R_o - R_i)} = \frac{1}{R_i}\left[\frac{W/n}{2\pi(R_o-R_i)}\right].$$

$$(p)_{x=R_o} = \frac{1}{150}\left[\frac{1750\times10^6}{6\times2\pi(200-150)}\right]$$

$$= 6.18\ kN/mm^2$$

$$(p)_{x=R_o} = \frac{1}{R_o}\left[\frac{W/n}{2\pi(R_o-R_i)}\right]$$

$$= 4.64\ kN/mm^2.$$

$$(p)_{x=R_m} = \frac{1}{\left(\frac{R_i+R_o}{2}\right)}\left[\frac{W/n}{2\pi\left(R_o-R_i\right)}\right]$$

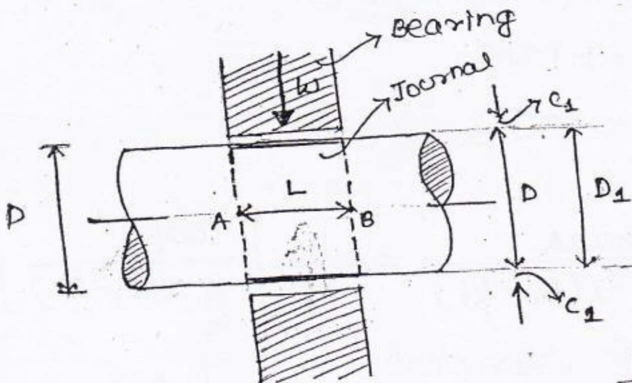
$$= 5.3\ kN/mm^2$$

Journal Bearings

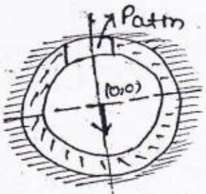
Journal bearing is a sliding contact radial bearing which is operating with hydrodynamic lubrication.

These bearings are suitable for in the applications where continuous service is required at high speed.

Eg :- I.C engine crankshaft
steam and gas turbine.
centrifugal pumps.
large size electric motors
Concrete mixturer



Journal bearing (or) Radial bearing



Bearings are always represented by inner dia. of circle.

D = Journal (or) shaft Dia.

D_1 = (Inner) Dia of Bearing

$$\boxed{D_1 = D + C}$$

C = Diametral clearance ~~ratio~~.

C_1 = Radial "

$$\boxed{C = 2C_1}$$

L = Length of the Journal (or) Bearing.

$$\boxed{\frac{C}{D} = \text{Diametral clearance ratio}}$$

$$\boxed{\frac{C}{D} = 0.001 \text{ to } 0.002} \Rightarrow$$ for satisfactory performance of Journal Bearing.

$p_{bearing}$ = bearing pressure

$$\boxed{p_{bearing} = \frac{W}{\underbrace{LD}_{\text{Projected area}}}}$$

$$p_{bearing} < p_{per}$$

$$\frac{W}{LD} \leq p_{per}$$

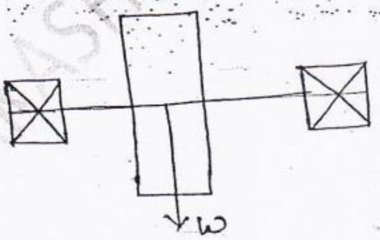
$$\boxed{L \geq \_\_\_\_ \text{ mm}}$$

$$\boxed{\begin{aligned} W_{max} &= \text{Strength of Journal Bearing} \\ &= LD\,p_{per}. \end{aligned}}$$

- Short bearing is treated as simple support.

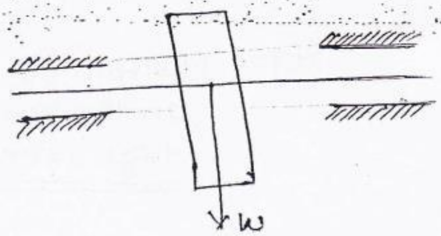
$$\frac{L}{D} = \frac{\text{Length of the bearing}}{\text{Dia. of the bearing}}$$

$\frac{L}{D} = 1 \Rightarrow$ Square bearing $\frac{L}{D} < 1 \Rightarrow$ short bearing	Act as Simple Support
$\frac{L}{D} > 1 \Rightarrow$ Long bearing $\Rightarrow$ fixed support	

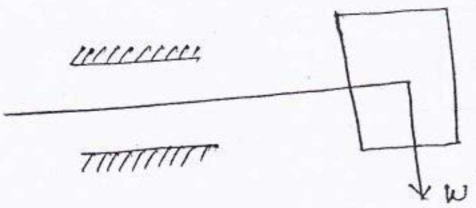
- 

S-S-B

$$\boxed{Y = \frac{WL^3}{48EI}}$$

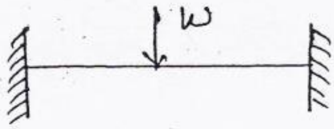
- 

$$\boxed{Y = \frac{WL^3}{192EI}}$$

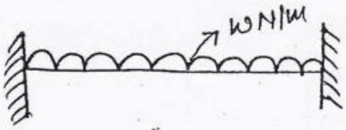
- 

Cantilever

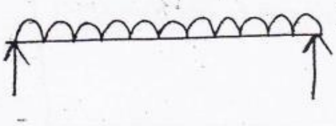
$$\boxed{Y = \frac{WL^3}{3EI}}$$

- 

$$\boxed{Y = \frac{WL^3}{192EI}}$$

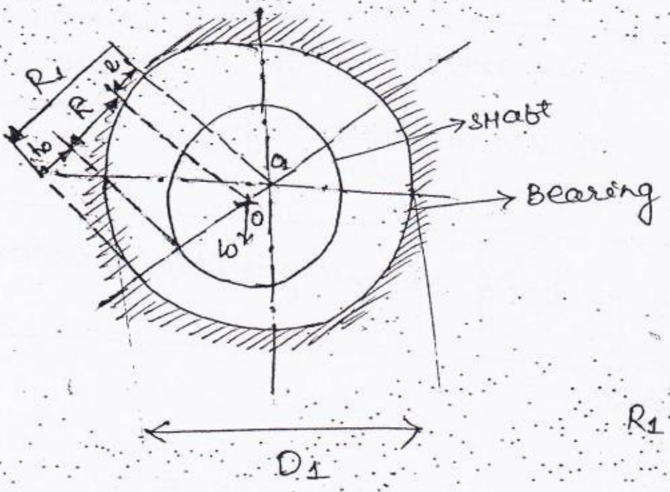
- 

$$\boxed{Y = \frac{wL^4}{384EI}}$$

- 

$$\boxed{Y = \frac{5}{384}\frac{wL^4}{EI}}$$

Eccentricity (e)



$R_1 \rightarrow$ Radius of bearing.

Fig:- Position of journal in the bearing at high speed

$$R_1 = e + R + h_o$$

$$e = R_1 - R - h_o$$

$$e = c_1 - h_o$$

$$\boxed{e = \frac{c}{2} - h_o}$$

Eccentricity ratio (ε)

It is ratio of eccentricity to radial clearance.

ε = Eccentricity ratio

$$\boxed{\varepsilon = \frac{e}{c_1}}$$

$$\boxed{\begin{aligned}\varepsilon = \frac{2e}{c} &= \frac{2}{c}\left[\frac{c}{2} - h_o\right] \\ &= 1 - \frac{2h_o}{c}\end{aligned}}$$

- If units are not mentioned in eccentricity (e) value then it is treated as eccentricity ratio.

Design procedure used in journal bearings

(1) Shaft or journal dia :- [D].

MSST

ASME $\Rightarrow$ $$\boxed{T_e = \sqrt{(K_b M)^2 + (K_t T)^2} = \frac{\pi}{16} D^3 \tau_{per}}$$

K_b and K_t are considered because of fatigue failure occur during bending.

How shaft is design?

If the B.M & T.M are equivalent then B.M eqn & T.M is equated.

steps for design (If B.M & T.M are unknown).

- Soderberg eqn.
- equivalent normal stress & equivalent shear stress are calculated.
- Then τ_{per} is calculated according to MSST & MDET.

$K_b \Rightarrow$ combined shock & fatigue factor for bending.

$K_t \Rightarrow$ Combined " " " factor for twisting.

(2) $D_1 = D + C = \_\_\_\_\_\_ mm$.

- Always inner dia of bearing is considered as the shaft is in contact with inner dia only.
- Dia can not be rounded as clearance can not be change.

(3) Length of bearing

$$p_{ind} \leq p_{per}$$

$$\frac{W}{LD} \leq p_{per}$$

$$L \geq \_\_\_\_\_\_ mm.$$

(4) P_{Loss} (or) Heat generated (H_g) :-

(power loss) $P_{Loss} = \mu \overset{N}{W} \overset{m/sec}{V} = \_\_\_\_\_\_$ watts

$$V = \frac{\pi \overset{m}{D} N}{60} = \_\_\_\_\_\_ m/sec.$$

- μ depends on following parameters :- $\frac{L}{D}$, $\frac{Zn}{P}$.

- Mc- kee's brothers

$$\mu \;\alpha\; f\left[\underbrace{\left(\frac{Zn}{p}\right)}_{\text{Bearing Characteristic no.}}, \left(\frac{D}{c}\right) \;\&\; \frac{L}{D}\right]$$

→ Dimension less.

$\frac{D}{C}$ → Reciprocal of dia. clearance ratio.

Z → Absolute viscosity of lubricant at an operating temp. of (t_0) lubricant

= ______ Pa-s (or) kgm/s (or) N-sec/m².

$$\boxed{1 \text{ centipoise} = 10^{-3} \text{ pa-s}}$$

n = speed of journal in r.p.s = r.p.m/60.

p = bearing pressure in Pa.

$$p = \frac{\underset{N}{W}}{\underset{m}{L}\ \underset{m}{D}} = \_\_\_\_ \text{ N/m}^2 \text{ (or) Pa}.$$

$$\frac{Zn}{p} = \frac{\cancel{Pa}-s}{\cancel{Pa}} \times \frac{1}{s} = \text{Dimensionless}$$

$$\boxed{\mu = \frac{33}{10^8}\left[\left(\frac{Zn'}{p'}\right)\left(\frac{D}{C}\right)\right] + K}$$

rpm

MPa

Depends on L/D ratio.

Leakage factor

$$\boxed{\begin{aligned} K &= 0.002 \text{ if } 0.75 \le \frac{L}{D} \le 2.8 \\ &= 0.003 \text{ if } \frac{L}{D} > 2.8 \end{aligned}}$$

- Power loss →

$$P_{loss} = H_g = \mu W V = \_\_\_\_ \text{ watts}.$$

(5) Heat dissipated (H_d)

$$H_d = C_d \times L D \times (t_b - t_a) = \_\_\_\_ \text{ watt}$$

$W/m^2\,°c$ m   m

C_d → Heat dissipation const coefficient

$= \_\_\_\_ \ W/m^2/°c$.

t_b → Bearing temp.

t_a → ambient or atm. temp.

(6) volume flow rate of coolant (V)

$H_g = H_d \Rightarrow$ Artificial cooling is not required.

$H_g > H_d \Rightarrow$ Artificial cooling is required.

$H_c = H_g - H_d =$ ______ watts.

$$H_c = \underset{kg/s}{m} \cdot \underset{J/kg/°c}{s} \left(t_{outlet} - t_{inlet} \right)$$

S = sp. heat of coolant = ______.

$m =$ ________ kg/sec.

$$\rho = \frac{m}{V}$$

$V = m^3/sec$

Imp

S = Sommerfield no.

** $\boxed{S = \left(\frac{Zn}{p}\right)\left(\frac{D}{c}\right)^2}$

$$\frac{Zn}{p}$$

$Z \to$ Pa.s, $n \to$ rps, $p \to$ pa

For a given bearing this no. is const.

M/c I	M/c II
$L_1 = 500mm$	$L_2 = 500mm$
$D_1 = 250\ mm$	$D_2 = 250mm$
$N_1 = 1000\ rpm$	$W_2 = 5\ kN$
$W_1 = 10\ kN$	$N_2 = ?$

$$S_1 = S_2$$

$$\left(\frac{Z_1 n_1}{p_1}\right)\left(\frac{D_1}{C_1}\right)^2 = \left(\frac{Z_2 n_2}{p_2}\right)\left(\frac{D_2}{C_2}\right)^2$$

$$\frac{n_1}{p_1} = \frac{n_2}{p_2} \Rightarrow \frac{1000}{p} = \frac{n_2}{p/2}$$

$$\boxed{n_2 = 500 \text{ rpm}}$$

Statement

For a given bearing sommerfield no. remains constant hence it is used to corelate the working conditions of different m/cs which are operating with the same bearing.

Expression for frictional torque in terms of viscosity of the lubricant

very imp **

$$\boxed{T_f = \frac{Z\pi^2 D^3 n' L}{60C}} \quad \text{— (I)}$$

$$\boxed{T_f = \mu W R} \quad \text{(II)}$$ If μ is given directly otherwise

(I) = (II)

$$\frac{Z\pi^2 D^3 n' L}{60c} = \mu W R$$

$$\Rightarrow \frac{Z\pi^2 D^3 n' L}{60C} = \mu W \frac{D}{2}$$

$$\Rightarrow \frac{Z\pi^2 D^3 n' \cancel{L}}{60C} = \mu \times p \times \cancel{L} \times \cancel{D} \times \frac{\cancel{D}}{2}$$

$$\mu = 2\pi^2 \left(\frac{Zn'}{60p}\right)\left(\frac{D}{c}\right)$$

PETROFF'S EQN (Not Actual eqn)

$$\boxed{\mu = 2\pi^2 \left(\frac{Zn}{p}\right)\left(D/c\right)}$$

- If $\left(\frac{L}{D} = k\right)$ is not given then use this eqn for μ.
- Actual eqn $\rightarrow$ Mc-kee's brothers eqn.

W.B (P-78)

(15) $\frac{\text{clearance}}{\text{radius}} = \frac{1}{100}$

$Z = 28 \times 10^{-3}$ Pa-s.

$n' = 2400$ rpm

$n = \frac{2400}{60} = 40$ rps.

$p' = 1.4$ MPa.

$p = 1.4 \times 10^6$ Pa.

$$\frac{C_1}{R} = \frac{C/2}{D/2} = \frac{1}{100}$$

$$\Rightarrow \boxed{\frac{D}{c} = 100}$$

$$S = \left(\frac{28 \times 10^{-3} \times 40}{1.4 \times 10^6}\right)(100)$$

$$= 8 \times 10^{-3}.$$

(16) $D = 50$ mm, $n' = 1450$

$n = \frac{1450}{60}$ rps

(radial clearance) $C_1 = 20$ micron $= 20 \times 10^{-6} \times 10^3$ mm.
$= 0.02$ mm.

$p' = 4$ N/mm^2.

$C = 2C_1 = 0.04$ mm.

$S = 0.0637$.

$Z = ?$, $0.0637 = \left(\frac{Z \times 24.16}{4 \times 10^6}\right) \times \left(\frac{50}{0.04}\right)^2$

$Z = 6.74 \times 10^{-3}$ pa-s

$Z = 6.74$ c.p [$\because$ 1 c.p $= 10^{-3}$ pa-s],

(17) $C = 0.020$ mm

17/01/14

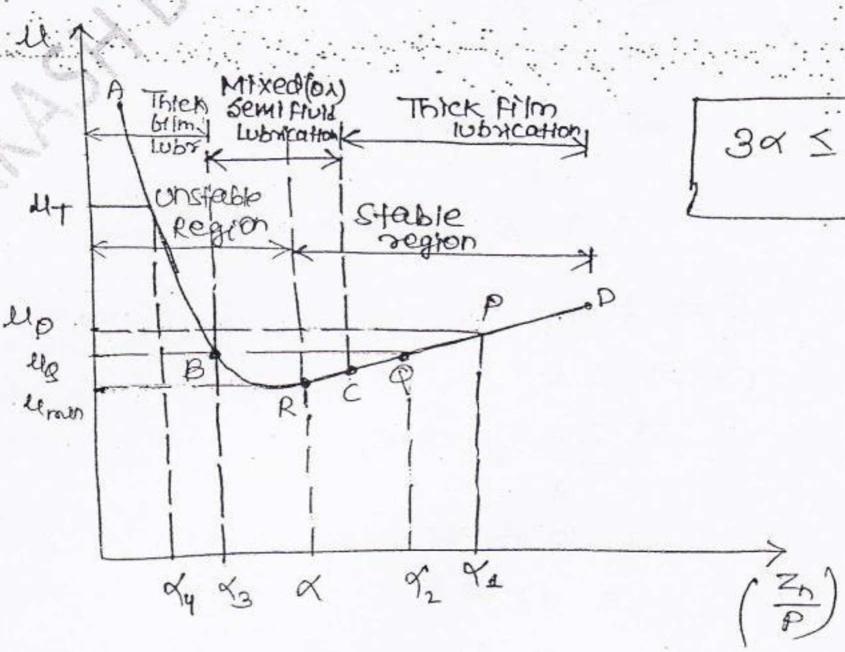
Significance of $\frac{Zn}{p}$:-

$\alpha \rightarrow$ Bearing modulus.

Value of Bearing characteristic no. corresponding to min. coefficient of friction

Safe condition

$$\boxed{\frac{Zn}{p} > \alpha}$$



$$\boxed{3\alpha \le \frac{Zn}{p} \le 15\alpha}$$

Stable region

$\alpha_1 \Rightarrow \mu_p$

$T\uparrow \Rightarrow Z\downarrow$

$\Rightarrow \frac{Z_n}{p}\downarrow$

$\Rightarrow$ P to Q

$\Rightarrow \mu\downarrow$

$\Rightarrow H_g\downarrow \Rightarrow T\downarrow$.

$\Rightarrow Z\uparrow$

$\Rightarrow \frac{Z_n}{p}\uparrow$

$\Rightarrow$ Q to P (i.e, it restores original cond^n).

$\alpha_1 \Rightarrow \mu_p$

$W\uparrow \Rightarrow p\uparrow$

$\Rightarrow \frac{Z_n}{p}\downarrow$

$\Rightarrow$ P to Q

$\Rightarrow \mu\downarrow$

$\Rightarrow T\downarrow$

$\Rightarrow Z\uparrow$

$\Rightarrow \frac{Z_n}{p}\uparrow$

$\Rightarrow$ Q to P.

$R \Rightarrow \mu_{min}$

$T\uparrow \Rightarrow Z\downarrow$

$\Rightarrow \frac{Z_n}{p}\downarrow \Rightarrow \alpha_3$.

$\Rightarrow S \Rightarrow \mu_s$

$\Rightarrow \mu\uparrow$

$\Rightarrow H_g\uparrow$

$\Rightarrow T \uparrow$

$\Rightarrow Z \downarrow$

$\Rightarrow \frac{Zn}{p} \downarrow (\alpha_4)$

$\Rightarrow \mu_f \uparrow$

$\Rightarrow H_g \uparrow$

$\Rightarrow T \uparrow \Rightarrow \frac{Zn}{p} \downarrow$

Hydrodynamic lubrication comes in stable region while Boundary " " " unstable region

For steady load conditions

$$\boxed{\frac{Zn}{p} = 3\alpha}$$

Highly fluctuating condition

$$\boxed{\frac{Zn}{p} = 15\alpha}$$

- $\frac{\text{change in viscosity}}{\text{change in temp.}}$ = viscosity Index = $\frac{dz}{dt}$

For thick film

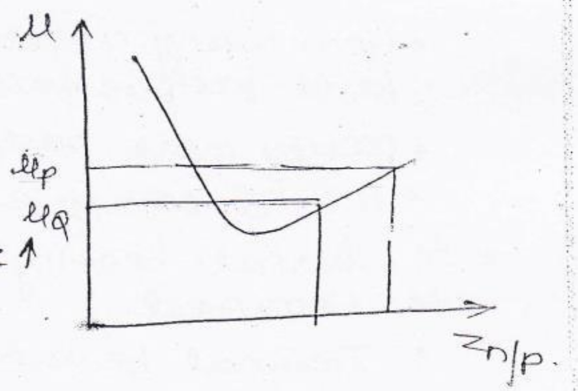
$W \uparrow \Rightarrow p \uparrow$

$\Rightarrow \frac{Zn}{p} \downarrow$

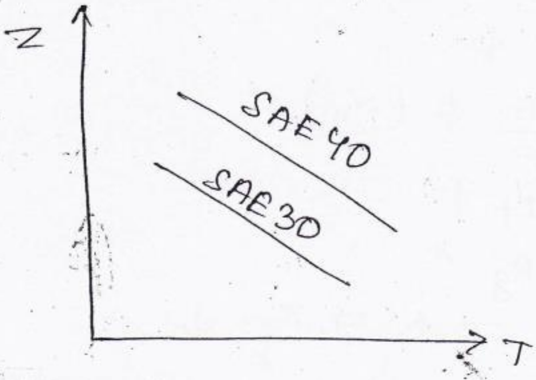
$\Rightarrow \mu \downarrow$

$\Rightarrow T \downarrow \Rightarrow Z \uparrow$

$\Rightarrow \frac{Zn}{p} \uparrow$



- $\frac{dz}{dt} \Rightarrow$ Measure of change in viscosity with change in temp.



(9) In thick film hydrodynamic journal bearing the coefficient of friction

(a) Increases with increase in load.

(b) Is independent of load.

(c) Decreases with increase in load

(d) May decrease or increase with load.
for mixed fluid.

Anti-friction bearing (A.F.B)

- They withstand both radial and thrust load.
- At higher speed vibrations are more, thus suitable for low speed (finite life).
- lubricant act as damper in case of sliding bearing which decreases vibration.
- How many revolution an A.F.B gone before first fatigue failure is called life of A.F.B.
- occupy more radial space.
- A.F.B occupy less axial space.
- In Journal bearing continuous lubrication is required.
- Journal bearing can withstand only radial load.
- Journal bearing cost is more, is suitable for intermittent service.
- A.F.B noise is more

- A·F·B are available with some Designation.
- A·F·B is studied on user the basis of user point of view not on designer pt. of view.

<u>Anti-friction bearing</u>

Procedure used in the selection of an AFB for a given application.

<u>A·F·B Designation</u>

SKF 6 3 06 → ×5 = 30mm → Shaft or bore dia.

6 → Type of AFB (6 → DGBB) Deep Groove Ball bearing

3 → Type of series.

(or)

(Equivalent to)

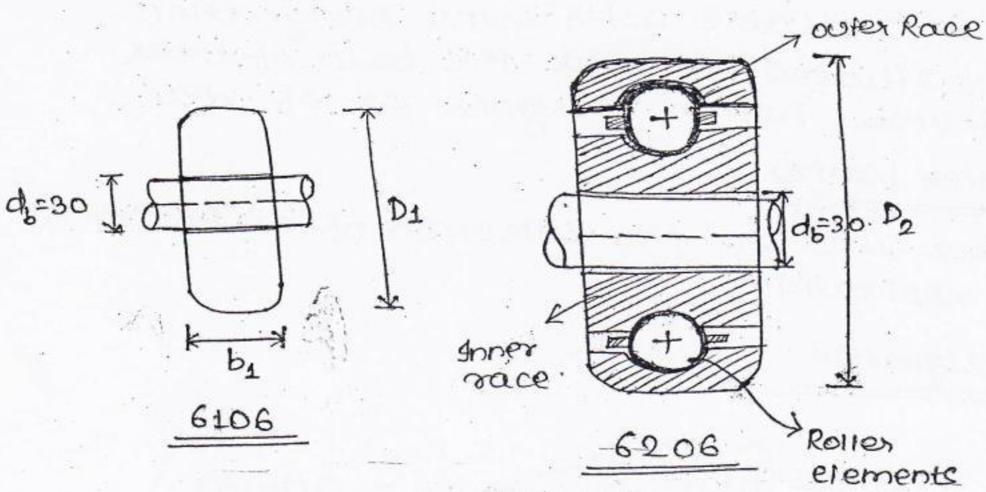
IS 30 BC 03 → Type of series.

30 → SHAFT DIA

BC → Type of AFB

SKF No. 6106 ⇒ 100 series (Extra light series)
SKF No. 6206 ⇒ 200 series (light series)
SKF No. 6306 ⇒ 300 series (Medium series)
SKF No. 6406 ⇒ 400 series (Heavy series)
SKF No. 6506 ⇒ 500 series (Extra series)

dimensions (b × Do)	C (KN)	Cost
INCREASES	INCREASES	INCREASES



Type of A.F.B → Based on characteristic.

Type of series → Based on dynamic capacity.

Rule is not valid for these Bearings.
SKF 6300 ⇒ 10
SKF 6301 ⇒ 12
SKF 6302 ⇒ 15
SKF 6303 ⇒ 17

$\boxed{\text{SKF} = 6304 \Rightarrow 20}$ from this (×5) rule is valid.

Parameters used in the selection of series of and Anti-friction bearing

(i) Equivalent load

As per AFBMA (AFB manufacturing association)

$$\boxed{P_e = S\left[XVF_r + YF_a\right]}$$ ⇒ if F_r, F_a & speed are const w.r.t time.

S = Service/shock factor

$S = 1 \Rightarrow$ steady loads

$S = 1 \Rightarrow$ Steady loads.
$= 1.5 \Rightarrow$ light shocks.
$= 2 \Rightarrow$ Moderate shocks.
$= 2.5 \Rightarrow$ Heavy "
$= 3 \Rightarrow$ Extra "

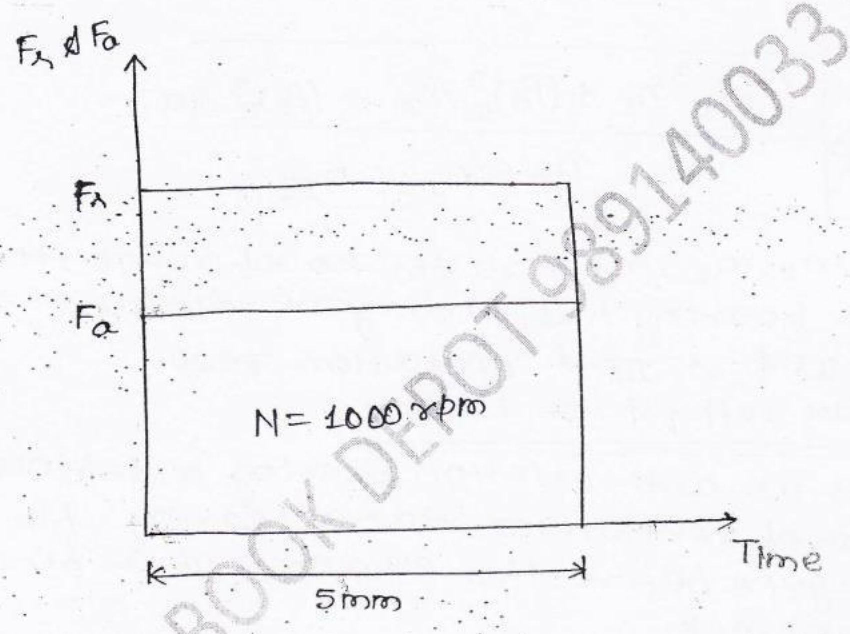
V = Race rotation factor
= 1 ⇒ Inner race rotation.
= 1.2 ⇒ outer " "

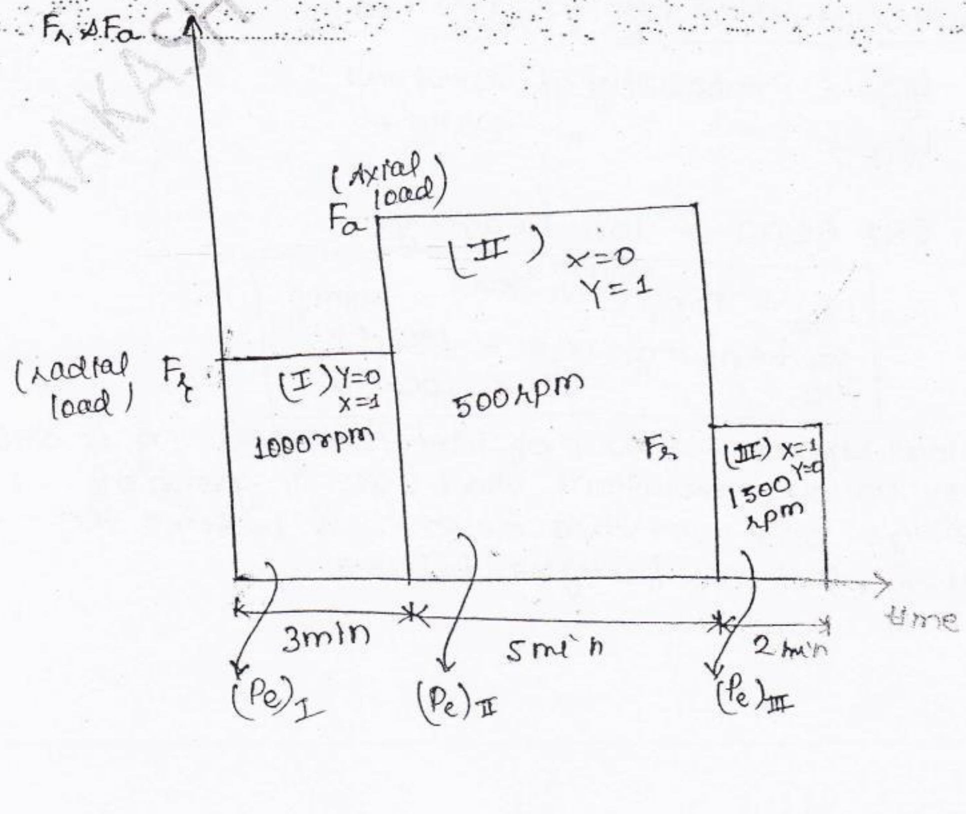
x = radial load factor.

Y = Axial load factor.

For thrust Ball bearing, $x=0$, $Y=1$

For cylindrical Roller bearing,
$x=1$, $Y=0$.





$$(P_e)_I = S[XVF_r) = 1\times1\times1\times F_r = F_r.$$

$$(P_e)_{II} = S[0+YF_a] = 1\times1\times F_a = F_a.$$

$$(P_e)_{III} = S[1\times1\times F_r] = F_r.$$

n_I = 3000 revolutions. [No. of revolns that bearings has gone during 1st, 2nd & 3rd operation]

n_{II} = 2500 revolutions.

n_{III} = 3000 revolutions

$$P_m = \sqrt[3]{\frac{(P_e)_I^3 n_I + (P_e)_{II}^3 n_{II} + (P_e)_{III}^3 n_{III}}{n_I + n_{II} + n_{III}}}$$

where n_1, n_2, n_3 are the no. of revolutions that a bearing has undergone during the I^{st}, 2^{nd} & III^{rd} operation resp.

Life of an anti-friction bearing

Life of an anti-friction bearing is defined as the no. of revolutions that a bearing has undergone before the evidence of a first fatigue crack.

Nominal or rated life [L_{90} (or) L_{10}]

$L_{90} \rightarrow$ Probability of survival

$L_{10} \rightarrow$ " " failure.

SKF 6206 $\rightarrow$ 100 bearings.

L_{90} = 100 million revolutions

90 bearings life $\geq$ 100 million revlns

10 " " < 100 mR.

Nominal life of a group of identical bearing is defined as the no. of revolutions that 90% of group of bearings can survive or exceed before the evidence of first fatigue failure.

Average life (L_{50})

→ probability of failure and probability of survival both are same.

$$\boxed{L_{50} = 5L_{90}}$$

No. of revolutions↑ ⇒ No. of failures ↑

⇒ No. of survival ↓

Life at any reliability other than 90%

$$** \quad \boxed{\frac{L}{L_{90}} = \left[\frac{\log_e\left(\frac{1}{R}\right)}{\log_e\left(\frac{1}{R_{90}}\right)}\right]^{1/1.17}}$$

$$\frac{L_{50}}{L_{90}} = \left[\frac{\log_e\left[\frac{1}{0.5}\right]}{\log_e\left[\frac{1}{0.9}\right]}\right]^{1/1.17} = 5$$

Basic Dynamic Capacity [C]

{Static capacity $[C_0]$}

Basic Dynamic capacity of a given antifriction bearing is defined as the no. of revolutions maxm load that 90% of the bearings can withstant for a minimum life 1 million revolution

SKF 6306

→ 100 bearings

→ C = 10KN.

Out of 100 bearings can withstand maxm load of 10KN.

$[P_e \text{ (or) } P_m] < C \Rightarrow L_{90} > 1 \text{ mr}.$

$[P_e \text{ (or) } P_m] = C \Rightarrow L_{90} = 1 \text{ mr}.$ (mr → million revolutions)

$[P_e \text{ (or) } P_m] > C \Rightarrow L_{90} < 1 \text{ mr}.$

objective

** $$\boxed{L_{90} = \left[\frac{C}{P_e \text{ (or) } P_m}\right]^K \text{ (or) } [\text{Loading ratio}]^K \text{ in mr}}$$

Numerator → max^m load that a bearing can withstand

If Num. = Den.

$L_{90} = 1 \text{ mr}.$

where,

$K = 3 \Rightarrow$ Ball bearing.

$= \frac{10}{3} \Rightarrow$ Roller bearing.

$$\boxed{\text{Load ratio} = \frac{\text{Dynamic capacity}}{\text{Equivalent (or) cubic mean load}}}$$

$\frac{1 \text{ yr at } 1000 \text{ rpm}}{P_e = 10\text{KN}} \Rightarrow L_{90} = \frac{365 \times 24 \times 60 \times 1000}{10^6} \text{ mr}$

$= 525.6 \text{ mr}.$

$P_e = 10\text{KN}$

$\Rightarrow C = ?$

$C = 80.7\text{KN}.$ (Minimum Capacity)

If options are given 80 & 82, we have to select 82.

Application

(1) Minimum of 6 months at 500 rpm.

(2) Dia. of shaft = 50mm.

(3) Noise should be minimum.

(4) $F_r = 10kN$, $F_a = 2kN$.

(5) $X = 1.5$, $Y = 0.5$.

Select on AFB

CONVENTIONAL

$$P_e = S[XVF_r + YF_a]$$

$$= 1[1.5 \times 1 \times 10 + 0.5 \times 2]$$

$$= 16\ KN.$$

$$L_{90} = \frac{6 \times 30 \times 24 \times 60 \times 500}{10^6}$$

$$= 129.6\ mr.$$

$$129.6 = \left(\frac{C}{16}\right)^3 \Rightarrow C = 80.96\ KN.$$

For selecting the bearing

SKF NO.	C (KN)
6110	25
6210	50
6310	80
6410	85
6310	100

For Pe

Parameter	I	II
F_r		
F_a		
N		

(Q) What is the condition for the load acting on a ball bearing if its life is to halfed.

Sol→ (load) $P_1 \rightarrow L_1$ (Life)

$$P_2 = ? \Rightarrow L_2 = \frac{L_1}{2}$$

for a given A.F.B,

$$L \propto \left(\frac{1}{P}\right)^K \quad (\because C = \text{const.})$$

$$\frac{L_1}{L_2} = \left(\frac{P_2}{P_1}\right)^k$$

$$\Rightarrow \frac{L_1}{L_{1/2}} = \left(\frac{P_2}{P_1}\right)^3 \Rightarrow P_2 = 2^{1/3} P_1$$

$$\Rightarrow P_2 = 1.26 P_1$$

Conclusion

If the load acting on a ball bearing increased by 26%, its life is reduced by 50%.

(Q) What is the life of a ball bearing if load acting on it reduced by 50%.

Sol→ $P_1 \rightarrow P$

$P_2 \rightarrow 0.5P$

$$\therefore \frac{L_1}{L_2} = \left(\frac{P_2}{P_1}\right)^k \Rightarrow L_1 = \left(\frac{0.5P}{P}\right)^k$$

$$\Rightarrow \frac{L_1}{L_2} = (0.5)^k \Rightarrow \frac{L_1}{L_2} = \left(\frac{1}{2}\right)^3$$

$$\Rightarrow L_2 = 8 L_1$$

Conclusion

If the load acting on a ball bearing becomes half of the original load then its life increases by 8 times than its original life

- If load as ~~as~~ well as life becomes double then bearing should be changed.

$$\boxed{\begin{array}{c} P_2 = 2P_1 \\ \hline L_2 = 2L_1 \end{array}}$$

- $$\frac{L_1}{L_2} = \left(\frac{C_1}{C_2} \times \frac{P_2}{P_1}\right)^3$$

$$\Rightarrow \frac{1}{2} = \left(\frac{C_1}{C_2} \times 2\right)^3 \Rightarrow C_2 = 2.5\, C_1.$$

Conclusion

If Load and life both are doubled then Capacity is increased by 2.5 times.

Practical applications of A.F.B

→ Machine tool spindle
→ Automobile front and ~~rare~~ rear axles.
→ Small size electric motor gear boxes.

(Q) A ball bearing is anticipated to have a life of 400 mr under an equivalent radial load of 10 KN with a reliability of 90%. the bearing

(a) If the load is 22 KN the life of the bearing with 90% reliability is [37.6 mr]

(b) with a reliability of 60% under the load 22 KN, the expected life is [Ans:- 145 mr].

(Q) The dynamic load capacity of 6306 bearing is 22 KN. The maxm radial load it can sustain to operate under 600 rpm is [Ans:- 5.29 KN].

Clutches

Friction clutches

Clutch is defined as a mechanical device which is used to engage / disengage the driven shaft to/from the driver shaft at the will of the operator, without stopping the prime mover.

Clutch is used to avoid frequent stopping and starting of the prime-mover.

Whenever driven m/c required intermittent service clutch is required.

~~Clutch is~~

Coupling	Clutch
Coupling are used to transmit power for both collinear and non-collinear shaft.	Always driven and driver shaft must be collinear. In automobile friction clutch are used.
Driven shaft speed	
(1) couplings are used in applications where driven m/c require continuous service.	(1) clutches are used in application where driven m/c require intermittent service.
• Couplings are used to transmit power b/w two collinear shafts and b/w two non-collinear shafts.	• Clutch is used to transmit power b/w two collinear shafts.
• Coupling is used to obtain a permanent connection b/w driver and driven shaft.	• Clutch is used to obtain a temporary connection b/w driver & driven shafts.
• In presence of coupling driver and driven shafts are rotating at the same speed.	• In presence of clutch driven shaft speed is variable and is not equal to the speed of driver shaft (A gearbox is provided).

① Prime mover $P = x\,kW$ → 1000 rpm → Gear Box $G = 2$ → Clutch $P = x\,kW$ → 500 rpm → Driven M/C

$T = \frac{P}{\omega} = T_2 > T_1$

← Better one ②

1 is not recommended.

② Prime mover $P = x\,kW$ → 1000 rpm → clutch → 1000 rpm → G.B $P = x\,kW$ → D.M/C

Receiving the power at high speed

$$P = T \cdot \omega$$

Here clutch directly receives the power thus Torque is more.

$$T_2 > T_1$$

$$T_2 \uparrow = \mu w \,[R_{eff} \uparrow]$$

In ① receive the torque after speed reduction.

when,

$$\omega \downarrow \Rightarrow T \uparrow \Rightarrow R_{eff} \uparrow \Rightarrow R_o \uparrow$$

Driven M/c 1 receives $P = x\,kW$ at 500 rpm

Driven M/c 2 " " at 500 rpm.

<u>Best location for gear box</u>

It is better to place the gear box b/w clutch and driven m/c, so that the clutch becomes economic.

CLUTCHES

- JAW CLUTCHES
  - Square Jaw clutches
  - Spiral jaw clutches
  - Saw tooth Jaw clutches
- FRICTION CLUTCHES
  - Axial friction clutches
    - Plate clutches
      - (Dry cluches) single plate clutches
      - Multi plate cluches (wet clutches)
    - Cone clutches
  - Radial friction clutches (centrifugal clutch)

Jaw clutches → M/c tools and rolling mills.

Single plate clutch → Four wheeler vehicles.

- In M.P.C, heat generated in large amount
- To overcome radial space problems, single plate clutch is replaced by multi-plate clutch.

If single plate clutch is replaced using multi-plate clutch then heat dissipation is major problem.

To overcome this coolant oil is required.

Due to this $\mu \downarrow \Rightarrow$ P.T.C $\downarrow$.

Centrifugal clutches (Automatic clutches) (used in mopeds).

Also called automatic clutches. Here, force require to engage the clutch acts along ⊥r axis of shaft.

In axial friction clutches

force is applied in axial dirn.

~~Axial friction clu~~

Radial friction clutches

Here, fore is applied in radial dirn (i.e, perpendicular to shaft axis)

Design equs used in plate clutches

Input Data

(1) P = X kW at Y rpm

(2) μ = ____

(3) Shaft dia. = ____

(4) P_{per} = ____

(5) Type of clutch (single plate or multi plate).

Properties of cluches

- High coeff. of friction.
- High thermal conductivity.
- High wear resistance.
- $\alpha \downarrow$
- Good strength.

Best for asbestos, ferodo, sintered metals.

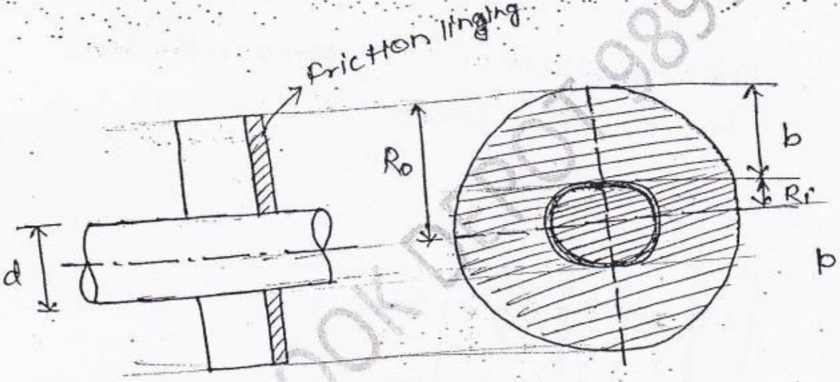
(1) T_F = frictional torque to be transmitted by the clutch.

$$T_F = \frac{\overset{\nearrow kW}{P} \times 60}{2\pi \underset{\searrow rpm}{N}} \times 10^6 = \_\_\_\_\_\_ \text{ N-mm}$$

- If not mentioned then assume all the given clutch is OLD (or) WORN OUT CLUTCH (U.W.T).
- for new clutches (U.P.T)

$R_i \rightarrow$ Inner radius of the friction lining / shaft radius.

$b \rightarrow$ width of friction lining



$$p = \frac{W}{2\pi r (R_o - R_i)}$$

Step after finding T_F

OLD OR WORN OUT CLUTCHES [UWT]

(2) R_o :-

$$(T_F)_{UWT} = n\mu W\left(\frac{R_o + R_i}{2}\right) = X$$

$$W = p_{per} \times 2\pi R_i (R_o - R_i)$$

$$\boxed{(T_F)_{UWT} = n\mu\pi p_{per} (R_o^2 - R_i^2) = X}$$

R_o = ______ mm

(3) b = width of friction lining.

$b = R_o - R_i$ = ______ mm.

(4) W = Axial force require to engage the clutch or spring force.

$$W = p_{per} \times 2\pi R_i (R_o - R_i)$$

New clutches (U.P.T)

(2) R_o :-

$$(T_F)_{UPT} = n\mu W\left[\frac{2}{3}\frac{[R_o^3 - R_i^3]}{(R_o^2 - R_i^2)}\right] = X \text{ N-mm}$$

$$W_{UPT} = p_{per} \times \pi (R_o^2 - R_i^2)$$

$$\boxed{(T_F)_{UPT} = n\mu\pi p_{per}\left[\frac{2}{3}(R_o^3 - R_i^3)\right] = X \text{ N-mm}}$$

R_o = ______ mm.

3) $b = R_o - R_i$ = ______ mm

4) $W = p_{per} \times \pi (R_o^2 - R_i^2)$ = ____ N.

n = no. of pair of frictional contact surfaces.

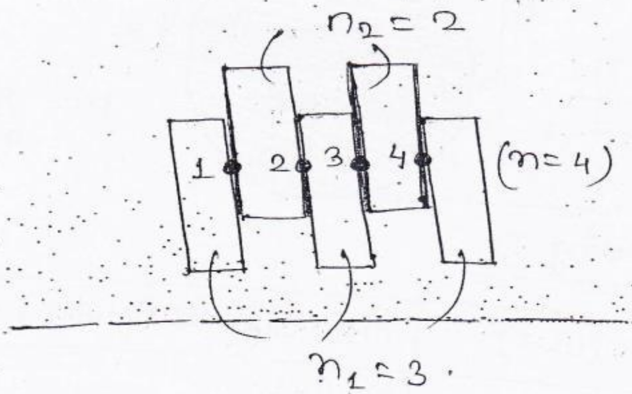
$n = 1 \Rightarrow$ s.p.c [single plate clutch] [Default].

$= 2 \Rightarrow$ s.p.c effective on both sides.

$\boxed{n = n_1 + n_2 - 1} \Rightarrow$ Multi-plate clutch (even no.)

n_1 = no. of plates (or) discs attached to driver shaft $= (n_2 + 1)$

n_2 = no. of plates (or) discs attached to driven shaft.



Ch→7

(5) $T_f = \frac{5 \times 60}{2\pi \times 2000} \times 10^6$

$= \underline{23873.2}$ N-mm.

$T_f = n \times \mu \times \boxed{P_{per} \times \pi (R_o^2 - R_i^2)} \times \frac{2}{3}\left[\frac{R_o^3 - R_i^3}{R_o^2 - R_i^2}\right]$

$\Rightarrow 1 \times 0.25 \times 1 \times \pi \times \frac{2}{3}(R_o^3 - 25^3) = 23873.2$

$R_o = 39.41$ mm.

(Q) clutch has outer and inner dia's 100mm and 40mm resp. Assuming uniform pressure of 2MPa and $\mu = 0.4$. Torque carrying capacity of the clutch is

Sol → $T_f = n \times \mu \times p_{per} \times \pi(R_o^2 - R_i^2) \times \frac{2}{3}\left[\frac{R_o^3 - R_i^3}{R_o^2 - R_i^2}\right]$

$= 1 \times 0.4 \times 2 \times \pi \times \frac{2}{3}\left(50^3 - 20^3\right]$

$= 196.03 \times 10^3$ N-mm

$= 196.03$ N-m.

(Q) A single plate clutch effective on both the sides carries a axial thrust of 1500N & effective radius of frictional surface 100mm and $\mu = 0.2$. The torque (in N-m) that can be transmitted.

Sol

$T_f = n \mu W R_{eff}$ [Both sides effective then $n = 2$]

$= 2 \times 0.2 \times 1500 \times \frac{100}{1000}$

$= 60$ N-m

(Q) A multi-plate clutch transmits 50kW of power at 1400 rpm. Axial intensity of pressure is not to exceed 0.15 MPa & $\mu = 0.12$, inner radius = 80mm = 0.7 outer Radius.

$(R_i = 0.7 R_o)$

Determine no. of plates to be mounted on driven & driven shafts.

Sol Pressure varying (UWT)

$T_f = n \mu \pi p_{per} (R_o^2 - R_i^2)$

$R_i = 80 = 0.7 R_o \Rightarrow R_o = 114.2$ mm

$T_f = \frac{P \times 60}{2\pi N} \times 10^6$

$$= \frac{50 \times 60}{2\pi \times 1400} \times 10^6$$

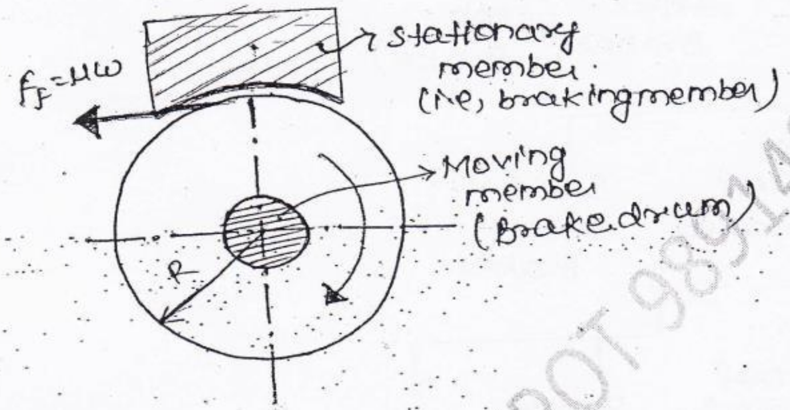
$$= 341046.3 \text{ N-mm.}$$

$$\therefore n = \frac{341046.3}{0.12\pi \times 0.15 \left(114.2^2 - 80^2\right)}$$

BRAKES

Brakes

Brake is a mechanical device which is used to retard the speed of moving member or to bring the moving member to a stationary condition in the state of rest. or to hold the body



Brake perform its f^n the above f^n by offering frictional resistance between a moving member i.e, brake drum and a stationary member (i.e, a braking member

$$T = \frac{P \times 60}{2\pi N}$$

$$T_F = \mu W R \, (\circlearrowright)$$

$$W \uparrow \Rightarrow T_F \uparrow$$

At particular value of W, T_F = Torque applied = Member is brought to stationary condition

For mechanical brake heat dissipation is major problem.

For railway, to bring to ~~to~~ stationary ~~to~~ it to stationary position mechanical brake is used.

BRAKES

- Mechanical Brakes
  - Radial Brakes
    - Block Brakes (SHOE)
      - (External) Block Brake
      - Internal expand shoe Brakes
    - Band Brakes
      - Simple Band Brake
      - Differential Band Brake
    - Band & Block Brakes
  - Axial Brakes (DISK Brake)
- Pneumatic Brakes
- Hydraulic Brakes
- Electrical Brakes.

ΔE = Energy to be absorbed by a braking member during braking action

$$\Delta E = \frac{1}{2} m(v_1^2 - v_2^2) + \frac{1}{2} I(\omega_1^2 - \omega_2^2) + mg(h_1 - h_2)$$

$1 \rightarrow$ Velocity and height before the application of brake.

$2 \rightarrow$ Vel. & height after the application of brake.

$\theta \rightarrow$ Angle turned by brake drum during Braking action

$$\Delta E = T_f \times \theta \Rightarrow T_f = ?$$

$t \rightarrow$ time.

$\alpha \rightarrow$ Acceleration.

$$\theta = \omega_1 t + \frac{1}{2} \alpha t^2.$$

$$\boxed{T_F = \mu R_N \times R}$$

$T_F \rightarrow$ frictional torque.

$T_F \times \omega = P$ $\qquad \omega \rightarrow$ Angular velocity

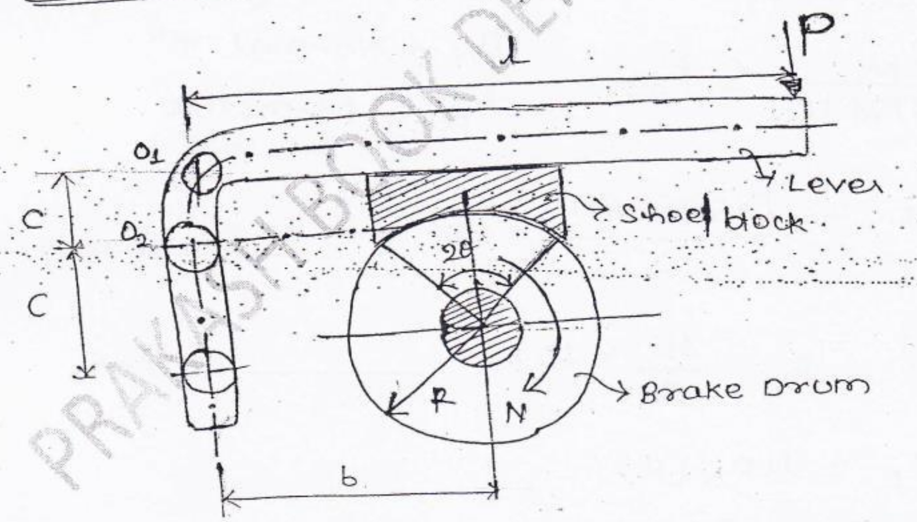
$\therefore T_F = \frac{P}{\omega}$

Here, $P = P_{absorption} \rightarrow$ Power absorption

R_N = Normal rkn.

Clutch	Brake
clutch is used to transmit power.	Retard the speed.
Before app. of clutch driven shaft is stationary & driver is moving	Both are brought to stationary position after application of Brake

Block Brake



C → Distance of the fulcrum from the line of action of frictional force.

O_2 → Fulcrum in line with the line of frictional ~~fotc~~ force

O_1 → Fulcrum above the line of action of frictional force

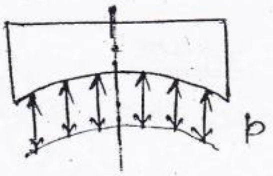
b → Fulcrum position from the centre of drum shoe is rigidly attached to the brak drum.

$2\theta \rightarrow$ Angle subtended by the shoe at the centre of shoe.

Shorta Shoe

low and medium power absorption.

long shoe $r\uparrow$, $T_F\uparrow$, size of shoes $\uparrow$, pressure is non-uniformly distributed (contact is less)



Short Shoe brake (pressure uniformly distributed throughout the contact)

contact dim$^n \rightarrow L\theta$.

$l \rightarrow R \times 2\theta$

$p_b \leq p_{per}$

$\frac{R_N}{R(2\theta)b} \leq p_{per}$

$p_b \rightarrow$ Bearing pressure

$R_N \rightarrow$ Normal rk^n.

$p_{per} \rightarrow$ permissible pressure

(1) $T_F = \frac{P_{absorption}}{\omega} \times 10^6 =$ ________ N-mm.

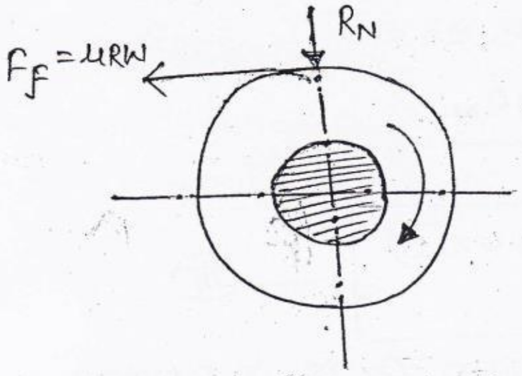
(or) $T_f = \frac{\Delta E}{\theta} \times 10^3 =$ ________ N-mm.

(2) $T_F = \mu R_N (R)$

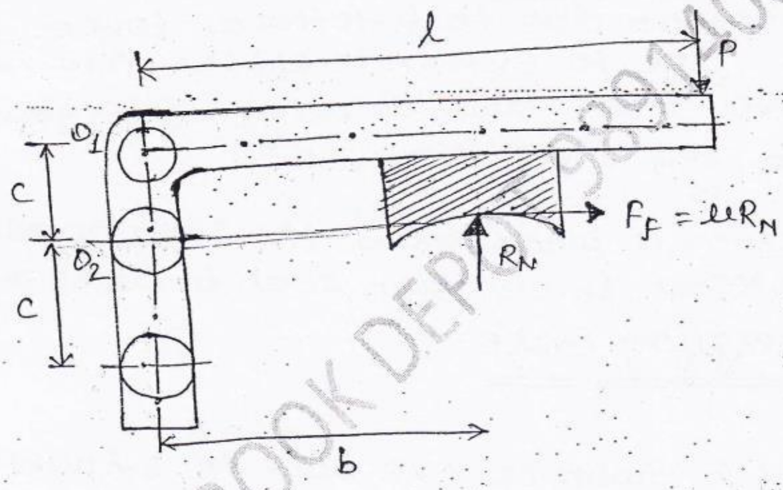
$R_N = ?$

(3) P:-

P is obtained by drawing F.B.D of lever $\Sigma M_o = 0$.



F.B.D of Drum



$\Sigma M_{O_1} = 0$

$$P \times l - \mu R_N (c) - R_N (b) = 0$$

$$\boxed{P = \frac{R_N b + \mu R_N c}{l}} \quad \text{—— (1)}$$

ΣM_{O_2}

$$\Rightarrow P \times l - R_N (b) = 0$$

$$\Rightarrow \boxed{P = \frac{R_N}{l}(b)} \quad \text{—— (II)}$$

$\Sigma M_{O_3} = 0$.

$\Rightarrow P \times l + \mu R_N (c) - R_N \times b = 0$

$\Rightarrow P = \dfrac{R_N b - \mu R_N c}{l}$

$\Rightarrow \boxed{P = \dfrac{R_N}{l}(b - \mu c)}$ —— (III)

For the given configuration O_3 is giving least effort.

Self energizing Brake

If the moment due to frictional force & moment due to external effort are in same dirn then such arrangement results in self energizing Brake.

If fulcrum is placed below the line of action of frictional force then such brakes are self energizing Brake.

Always P should be +ve but ~~m~~ should be as minimum as possible.

Statement

For the given configuration of the block brake (CW) fulcrum O_3 is the best because it gives least effort (i.e, self energizing brake).

A brake is said to be a self energizing if the dirn of the moments of frictional force and external effort are in the same dirn with respect to the fulcrum.

Always the fulcrum ~~should~~ should be positioned in such a way that a brake should be a self energizing.

Self energizing brake should be designed in such a way that the external effort should be +ve and as minimum as possible.

from eq^n(III)

case (I) :- $(b > \mu c)$

$\Rightarrow P = +ve$

case II :- $b = \mu c$

$\Rightarrow P = \text{zero}$

$\Rightarrow$ self locking brake.

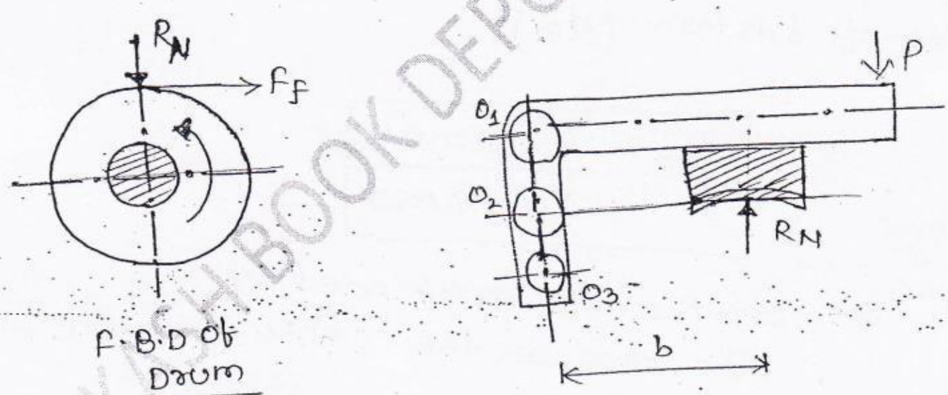
case III :- $b < \mu c$

$\Rightarrow P = -ve$

$\Rightarrow$ uncontrollable Braking action.

- If the drum is rotating in ACW dirn then O_1 is the best fulcrum.

when drum is rotating in ACW dirn



F.B.D of Drum

$\Sigma M_{O_1} = 0$

$P \times l + \mu R_N (c) - R_N (b) = 0$

$$\boxed{P = \frac{R_N}{l}[b - \mu c]} \quad \text{— (I)} \quad \Rightarrow \text{(self-energizing brake)}$$

$\Sigma M_{O_2} = 0$

$\Rightarrow P \times l - R_N (b) = 0$

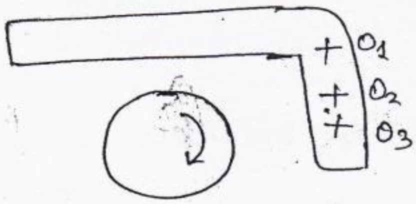
$$\Rightarrow \boxed{P = \frac{R_N (b)}{l}} \quad \text{— (II)}$$

$\Sigma M_{O_3} = 0$

$\Rightarrow P \times l - \mu R_N (c) - R_N (b) = 0$

$$\Rightarrow \boxed{P = \frac{R_N}{l}(b + \mu c)} \quad \text{— (III)}$$

- Frictional force is itself sufficient when no external force is applied to keep the lever in equilibrium position,



- For the given conf. O_1 is best.

- For the given configuration of the block brake anticlockwise rotation of the drum, fulcrum O_1 is the best because it use a self - energizing brake.

Analysis of long shoe brakes (i.e, when pressure is non-uniformly distributed]

It is similar to short shoe brake when actual coeff. of friction (μ) is replaced by equivalent coeff of friction (μ_e).

$$\boxed{\mu_e = \frac{4\mu \sin\theta}{2\theta + \sin 2\theta}}$$

θ = semi-angle subtended by the shoe at the centre of drum,

$2\theta \rightarrow$ should be in radian.

$p_{ind} \leq p_{per}$

$$\frac{R_N}{lb} \leq p_{per}$$

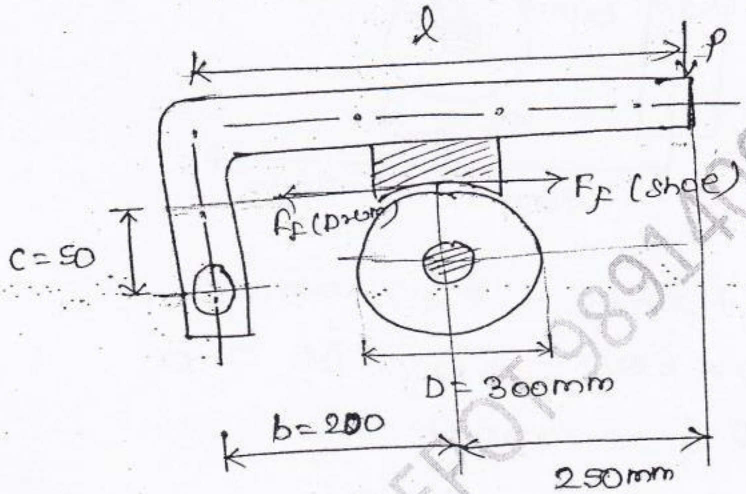
$$\boxed{\begin{array}{l} \dfrac{R_N}{R(2\theta)(b)} \leq p_{per} \\ \Rightarrow b \geq \_\_\_\_ \text{ mm} \end{array}}$$

(Q) A single block brake with 300 mm dia shown in the fig. is used to absorb a torque of 75 N-m, $\mu = 0.35$, the pressure on the block is uniform and calculate the force P.

Sol:– $D = 300$ mm

$T_F = 75$ N-m, $\mu = 0.35$

$$P = \frac{R_N}{l}(b + \mu c)$$



R → Radius.

$$T_F = \mu R_N \times R$$

$$75 \times 10^3 = R_N \times 150 \times 0.35$$

$$\Rightarrow R_N = 1428.57 \text{ N}$$

$$P = \frac{1428.57}{4500}(250 + 0.35 \times 50)$$

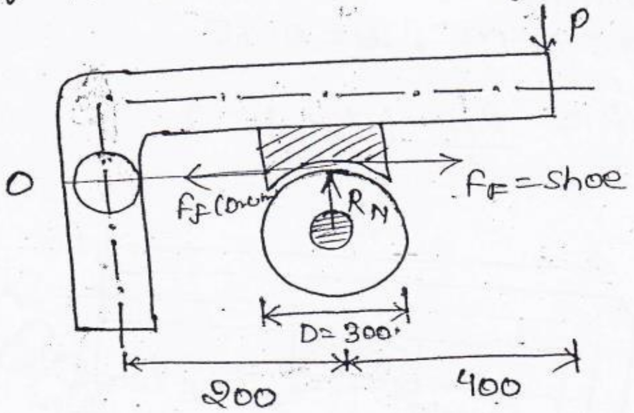
$$\Sigma M_O = 0$$

$$\Rightarrow P \times 450 + (\mu R_N) \times c - R_N(200) = 0$$

$$\Rightarrow P = \frac{1428.57 \times 200 - 0.35 \times 1428.57 \times 50}{450}$$

$$\Rightarrow P = 579.36 \text{ N}.$$

(9) A block brake shown below has a face width of 300 mm and mean coeff. of friction of 0.25 for an activating force of 400N, the braking torque in N-m



$$\Sigma M_O = 0 \Rightarrow P \times 600 - R_N \times 200 = 0$$

$$400 \times 600 - R_N \times 200 = 0$$

$$\Rightarrow R_N = 1200\,N$$

$$T_f = \mu R_N \times R$$

$$= 0.25 \times 1200 \times 150$$

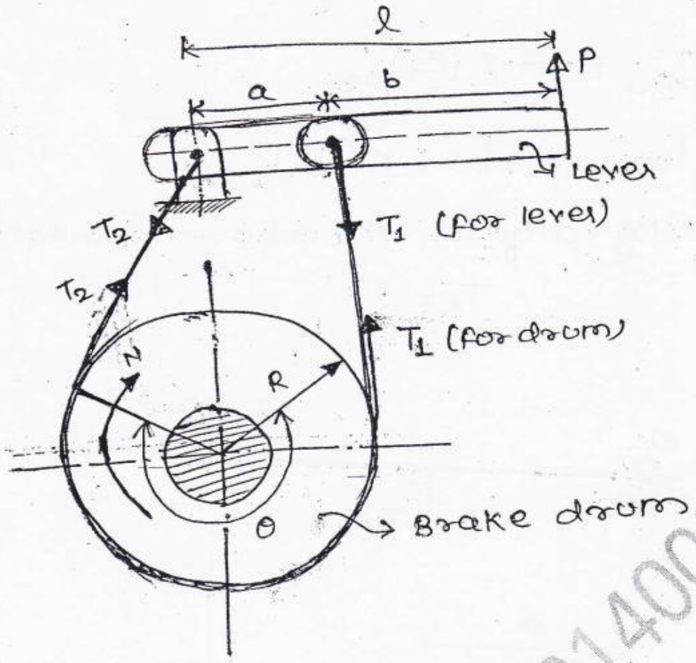
$$= 45\ N\text{-}m$$

Band Brakes

If not mentioned any thing assume simple band brake.

Simple band	Differential band brake
when one end of band ~~fulcrum~~ passes through fulcrum.	when two ends of the band attached to the fulcrum. Dir^n must be given in this case.

Anticlockwise rotation gives the least effort.



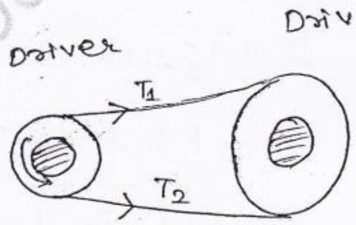
(1) $T_f = \dfrac{P_{absorption}}{\omega}$

(or) $\dfrac{\Delta E}{\theta} =$ —— N-mm

(2) $T_f = (T_1 - T_2)R$

$\Rightarrow T_1 - T_2 =$ —— N —(I)

(3) $\dfrac{T_1}{T_2} = e^{\mu\theta}$ → radians —(II)



Entering portion → tight side
leaving portion → loose (slack) side

By solving these two eqns T_1 & T_2 can be calculated.

If θ is not given, then

$$(\sigma_t)_{max} \le (\sigma_t)_{per}$$

$$\frac{T_1}{b \times t} \le (\sigma_t)_{per}$$

$$T_1 \le \boxed{b \times t\,(\sigma_t)_{per}} \rightarrow T_{max}$$

$$T_{max} = A \times (\sigma_t)_{per}$$

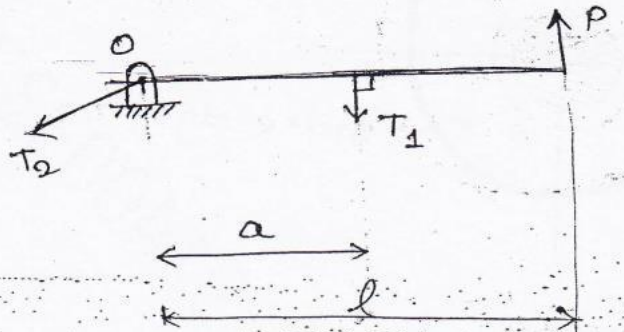
$$T_{max} = b \times t \times (\sigma_t)_{per}$$

By using any of the above two eqns,

$T_1 = ?$

$T_2 = ?$

(5)

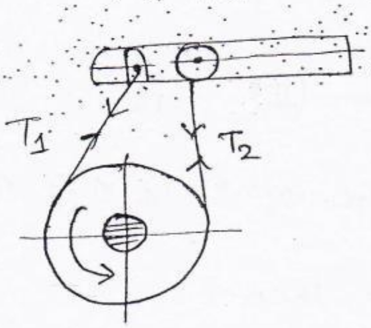


$\Sigma M_0 = 0$

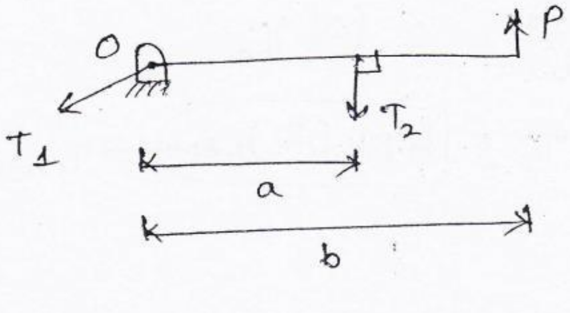
$\Rightarrow -P \times l + T_1 \times a + T_2 0 = 0$

$\Rightarrow \boxed{P = \frac{1}{l}(T_1 a)}$

- In simple band brake there is no possibility of self locking.
- In simple band brake one end of the band passes through the fulcrum.



All calculations are same only ΣM calculation changes.



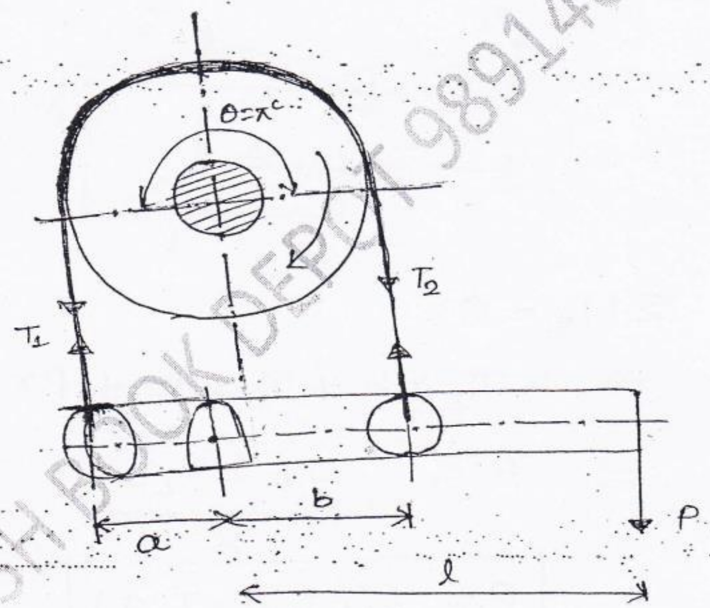
$$\Sigma M_0 = 0 \Rightarrow -P \times l + T_2 a + (T_1)(0) = 0$$

$$\Rightarrow \boxed{P = \frac{1}{l}(T_2 a)}$$

- For the given calculation of band brake the best configuration is ~~AC~~ Anticlockwise rotation because less effort is required.

Differential band brake

When both the ends of band is attached to the fulcrum.



(1) $T_f = \frac{P_{absorption}}{\omega}$ (or) $\frac{\Delta E}{\theta}$ = ____ N-mm.

(2) $T_f = (T_1 - T_2)R \Rightarrow T_1 - T_2 =$ ____ N —(I)

(3) $\frac{T_1}{T_2} = e^{\mu\theta \rightarrow radian} \Rightarrow \frac{T_1}{T_2} =$ ____ (II)

(4) $(\sigma_t)_{max} \leq (\sigma_t)_{per}$

$$\frac{T_1}{b \times t} \leq (\sigma_t)_{per}.$$

$$T_1 \le \underbrace{b \times t \times (\sigma_t)_{per}}_{T_{max}}$$

$$T_{max} = A \times (\sigma_t)_{per}$$

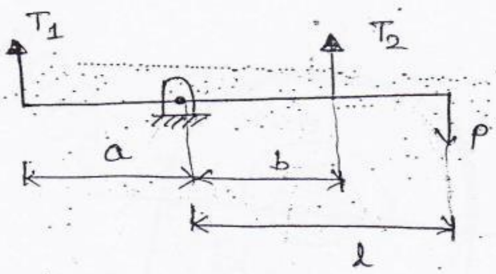
$$T_{max} = b \times t \times (\sigma_t)_{per} \quad \text{--- (III)}$$

By using any of the above two eqⁿˢ,

$T_1 =$ —

$T_2 =$ —

(5) <u>P :—</u>



$$\Sigma M_0 = 0$$

$$\Rightarrow \quad -T_2 \times b + T_1 \times a + P \times l = 0$$

$$\Rightarrow \quad P = \frac{T_2 b - T_1 a}{l}$$

$$\boxed{P = \frac{1}{l}(T_2 b - T_1 a)}$$

To avoid self locking,

$$P > 0 \Rightarrow T_2 b - T_1 a > 0$$

$$\Rightarrow T_2 b > T_1 a$$

$$\boxed{\frac{b}{a} > \frac{T_1}{T_2}}$$

- fulcrum is nearer to tight side.
- Self locking is possible here,

$$\boxed{\frac{b}{a} = \frac{T_1}{T_2}}$$

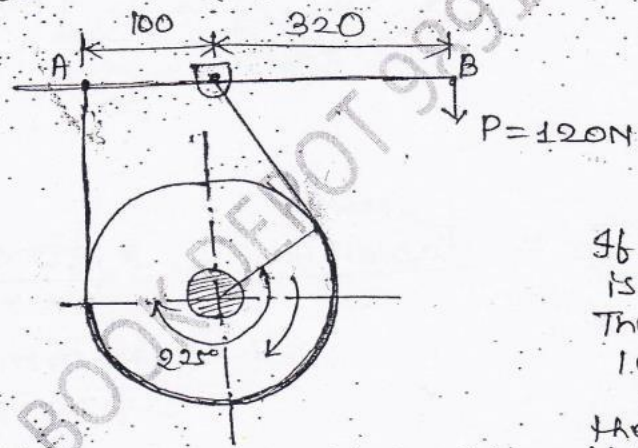
(Q) In a band brake shown in fig. if it is designed for the self locking condition with a ratio of tensions 2.2 then the the ratio $\frac{a}{b}$ is

Sol :- $\frac{b}{a} = \frac{T_1}{T_2}$

$\Rightarrow \frac{a}{b} = \frac{1}{2.2} = 0.45.$

(Q) The band brake as shown in the fig. is applied to a shaft carrying a flywheel of mass 450 kg. The drum dia. is 250 mm and $\mu = 0.25$. The Torque applied on the drum is

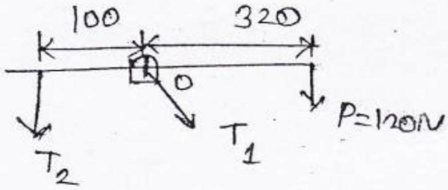
Sol



If dirn of rotation is not given Then take the least effort side means the end passes through fulcrum should pass treated as tight side.

Passing through the fulcrum thus simple band brake

$T_f = (T_1 - T_2) R$



$\frac{T_1}{T_2} = e^{0.98}$

$= 2.66.$

$\Sigma M_O = 0$ $\quad 120 \times 320 - 100\, T_2 = 0$

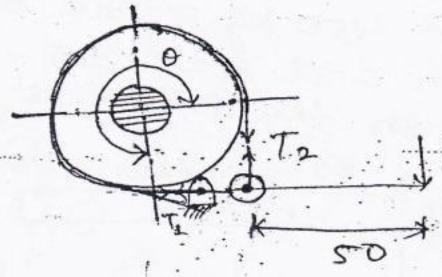
$\Rightarrow T_2 = 384$ N -

$T_1 = 384 \times 2.66 =$

Note

If the dir^n of rotation of drum is not given then ~~the~~ to get the least effort always tight side of the Band should pass through the fulcrum.

(Q) A Band brake below absorb 30kW, when a drum rotates at 500rpm. Dia of Drum is 500mm, $\mu = 0.3$, permissible tensile stress of band material is 80 MPa. Then width of band is.

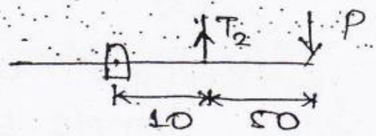


Tight side $T_2 \to$ Pass through fulcrum.

$$T_f = \frac{P_{absorption}}{\omega} = \frac{\cancel{P}\, 30 \times 60 \times 10^3}{2\pi \times 500}$$

$$= \cancel{4.58 \times 10\ N\text{-}mm} = 572.95 \times 10^3\ N\text{-}mm$$

$$= 573 \times 10^3\ N\text{-}mm$$



$$T_f = (T_1 - T_2) R$$

$$T_1 - T_2 = \frac{T_f}{R} = 2.29 \times 10^3 = 2292$$

$$\frac{T_1}{T_2} = e^{0.3 \times 3\pi/2} = 4.11$$

$$T_1 = T_2 \times 4.11$$

$$\Rightarrow 4.11 T_2 - T_2 = 2.29 \times 10^3$$

$$\Rightarrow T_2 = \cancel{557.47}\ 736\ N.$$

$$T_1 = 3028.9\ N.$$

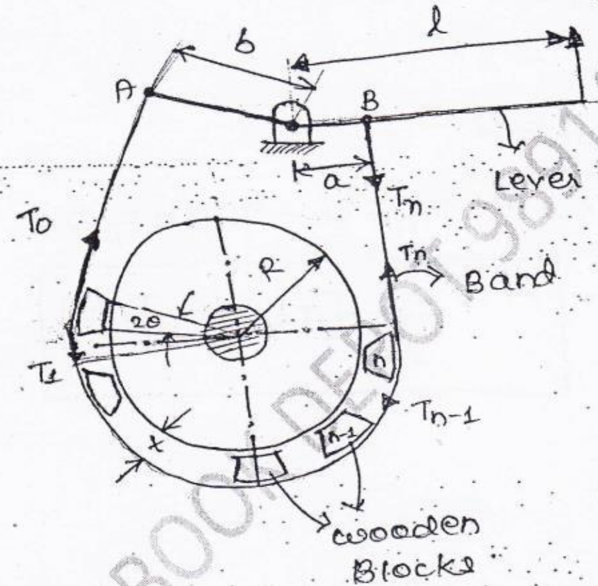
(9) Repeat the above Q. for the effort to be applied on the lever

$$P \times 60 - T_2 \times 10 = 0$$

$$\Rightarrow P = \frac{3828.9 \times 10}{60} \quad \frac{736 \times 10}{60}$$

$$\Rightarrow P = 122.6 \text{ N}$$

Band and Block Brakes



$2\theta \rightarrow$ Angle subtended by each block at the centre of drum.

$$\frac{T_1}{T_0} = \frac{1+\mu \tan\theta}{1-\mu \tan\theta}$$

$$\frac{T_n}{T_0} = \frac{T_1}{T_0} \times \frac{T_2}{T_1} \times \frac{T_3}{T_2} \times \text{---} \times \frac{T_{n-1}}{T_{n-2}} \times \frac{T_n}{T_{n-1}}$$

$$\boxed{\frac{T_n}{T_0} = \left(\frac{T_1}{T_0}\right)^n = \left(\frac{1+\mu \tan\theta}{1-\mu \tan\theta}\right)^n}$$

$\theta \rightarrow$ semi-angle subtended by each block at the centre of drum.

Braking Torque

$$T_F = (T_n - T_o)(R+t)$$

$$T_n - T_o = \underline{\qquad\qquad} \rightarrow \text{I}$$

$$\frac{T_n}{T_o} = \left(\frac{1+\mu\tan\theta}{1-\mu\tan\theta}\right)^n = \underline{\qquad\qquad} \rightarrow \text{II}$$

$T_n = ?$

$T_o = ?$

$$\Sigma M_o = 0$$

$$-P \times l + T_n \times a - T_o \times b = 0$$

$$\Rightarrow \boxed{P = \frac{1}{l}\left[T_n a - T_o b\right]}$$

Design of spur gear

⇒ To determine module (m) of a Gear Tooth.

⇒ Module (m) is determined by Lewis beam strength eqn.

⇒ Determine the dimensions of gear tooth like

$$D = m \cdot Z$$

Z → No. of teeth,
D → Diameter.

Circular pitch, $p_c = \pi m$,
Diameter " , $p_d = \frac{1}{m}$

(a) Addendum = m,
(d) deddendum = 1.157m

Clearance → Diff. of add. & dedd. of two m/c gear tooth.
$= d - a = 0.157m$

• 1 → Smaller → Pinion (Driver gear)
• 2 → gear → Driven.

Face width,

$$\boxed{b = 10\,m}$$

Centre Distance,

$$\boxed{C.D = \frac{D_1 + D_2}{2} = \frac{m}{2}(Z_1 + Z_2)}$$

⇒ to determine beam strength of the gear tooth (F_s)

⇒ Determination of Dynamic load (F_d)

$\boxed{F_d \le F_s}$ ⇒ Design is safe w.r.t bending failure

⇒ Determination of wear strength (F_w)

$$\boxed{F_d \le F_w}$$

⇓

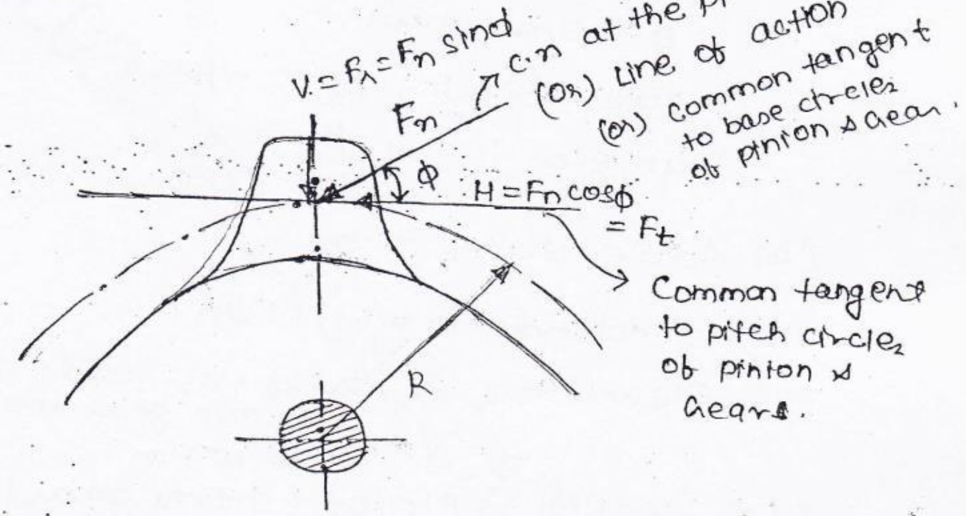
Design is safe w.r.t wear failure.

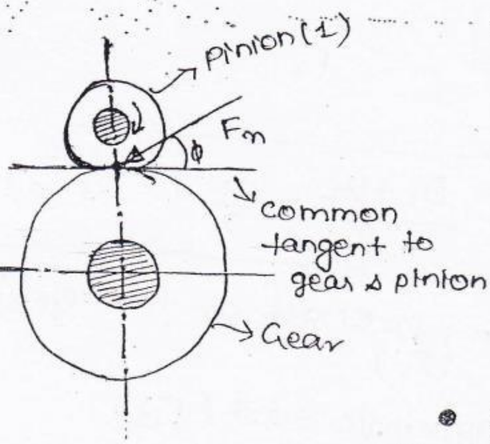
Beam strength

Max^m tangential load

F_d ⇒ Actual tangential load.

FORCE ANALYSIS ~~OF~~ IN SPUR GEARS





Vertical gear drive

- ϕ → Pressure angle
- F_n → Common normal (to pinion & gear) at the pt. of contact. Common
- Pinion → Driver
- Gear → Driven
- F_n depends upon the dir^n of rotation of pinion.

$$\boxed{\begin{array}{l} F_t = F_n \cos\phi \\ F_r = F_n \sin\phi \ (\text{or}) \ F_t \tan\phi \\ F_n = \sqrt{F_t^2 + F_r^2} \end{array}}$$

$\cos\phi >> \sin\phi$

($\because \phi$ is small)

$$\boxed{F_t >> F_r}$$

$F_r <<< F_t$

Hence,

Effect of F_r is neglected

M/C elements → / Forces ↓	Gear Tooth	SHAFT
F_r	Axial comp. Load	T.S.L
F_t	T.S.L	E.T.S.L ($e = R$)

Gear tooth is designed on the basis of bending stress.

Extremer fibre are subjected to axial & bending stress.

Inner fibre → All three.

Because bending stress design is based on maxm bending stress.

Gear tooth is subjected to → Axial comp.; shear & bending stresses.

In the design of Gear tooth → axial comp. stresses are neglected.

Gear tooth is based on max^m bending stresses.

For shaft,

$$F_{t_1} = F_t \Rightarrow H\text{-}T\text{-}S\text{-}L$$

One Bending moment in Horizontal plane and one shear force.

$F_r \Rightarrow$ vertical T·S·L

one B·M & one S·F (vertical plane)

F_t & F_{t_2}

$$\boxed{T_M = F_t \times R}$$

F_t is responsible for power transmission.

(i) $T = \dfrac{P \times 60}{2\pi N} \times 10^6 =$ ______ N-mm.

(ii) $F_t = \dfrac{T}{R}$ (or) $\dfrac{2T}{D \text{ (or) } mz}$

D → Torque acting on pinion or gear that D is taken (which r.p.m is given torque depends on that r.pm).

(iii) $F_r = F_t \tan\phi$

(iv) $F_n = R = \sqrt{F_t^2 + F_r^2}$

$$F_{t_1} = F_{t_2} = F_t = \frac{T}{R} \text{ (or) } \frac{2T_1}{(D_1 \text{ (or) } mz_1)} = \frac{2T_2}{D_2}$$

$$F_{r_1} = F_{r_2} = F_r = F_t \tan\phi$$

$$F_{n_1} = F_{n_2} = F_n = \sqrt{F_t^2 + F_r^2}$$

19|01|14

(22) $T_1 = \frac{P}{2\pi N_1} \times 10^6 = \frac{20}{2\pi \times 30} \times 10^6 = 106.103 \times 10^3 N\text{-mm}$

$F_t = \frac{2T_1}{D_1}$ (or) $\frac{2T_2}{D_2}$

$F_t = \frac{2 \times 106.103 \times 10^3}{5 \times 20} = 2122.06\ N.$

$F_r = F_t \tan\phi = 772.36 N$

$R = F_n = \sqrt{F_t^2 + F_n^2} = 2258.24\ N.$

(21) $C = \frac{D_1 + D_2}{2}$

$C = \frac{m}{2}(z_1 + z_2) = \frac{5}{2}(40 + 20).$

$= 150mm.$

(22) Arc of contact

$= \frac{\text{Length of path of contact}}{\cos\phi} = \frac{19}{\cos 20°}$

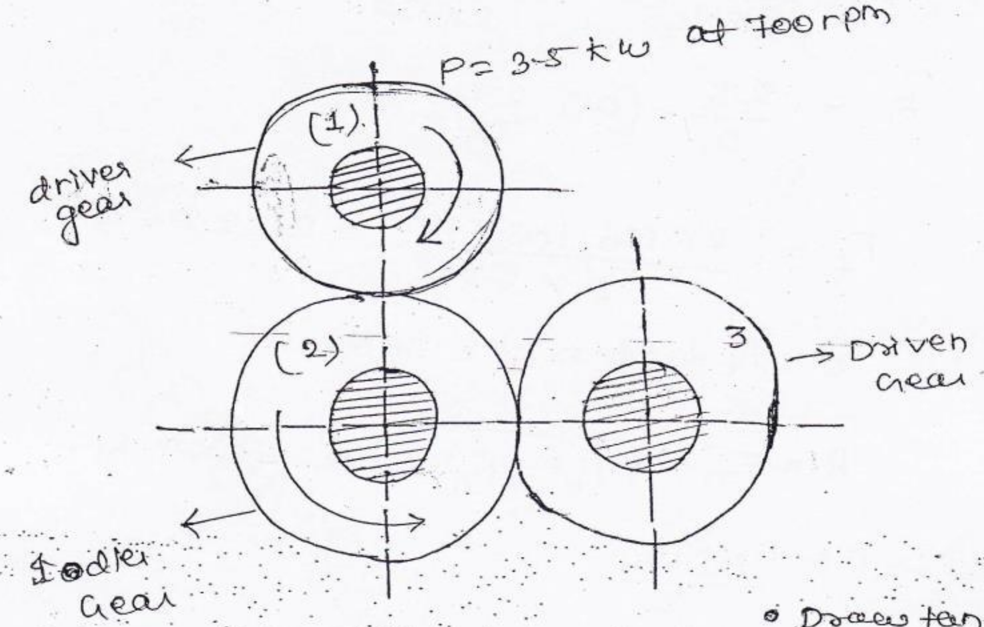
$= 20.21$

Contact ratio $= \frac{\text{Arc of contact}}{P_c} = \frac{20.21}{\pi \times 5}$

$= 1.28.$

(9) For the gear train as shown in the fig. Determine resultant force acting on the idler gear shaft as shaft as shown in the fig. Assume module = 5mm and pressure angle for all gears is 20°, no. of teeth on driver, idler and driven gear are 30, 60, 40 resp.

Idler gear → Gear which does not transmit power.



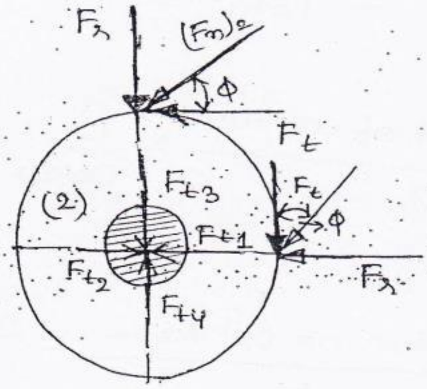
- Draw tangent at the contact pt.
- Then draw dirn of force f^n along the dirn of rotation 1 is rotating w.r.t 2 thus this Fn is $(F_n)_2$.
- Fn is equal as same power is transmited. when Fn is same the Ft & Fr are also same as pressure angle is same.

$$\boxed{P_{input} = P_{output}}$$

$\because \eta = 100\%$

$F_{n_1} = F_{n_2}$

$\left.\begin{array}{l} F_{t_1} = F_{t_2} \\ F_{r_1} = F_{r_2} \end{array}\right\} (\phi)$



$H = F_t + F_r$

$V = F_r + F_t$

$R = \sqrt{H^2 + V^2} = \sqrt{2}\,[H \text{ (or) } V]$.

F_{t_2} and F_{t_1} are dummy loads w.r.t. horizontal forces ~~bx~~ and F_{t_3} and F_{t_4} are dummy loads w.r.t vertical loads.

Why idler gear does not transmit power?

$F_{t1} = F_{t2} = F_t$

$F_{t3} = F_{t4} = F_t$.

$F_t \& F_{t2} \Rightarrow (T.M)_1 = F_t \times r$ (↻)

$F_t \& F_{t4} \Rightarrow (T.M)_2 = F_t \times r$ (↺)

Net T.M

$= (T.M)_2 - (T.M)_1 = 0$.

- Idler gear is designed based on only B.M eqn.

$$T = \frac{P \times 60}{2\pi N}$$

$$T = \frac{3.5 \times 10^3 \times 60}{2\pi \times 700} \times 10^3$$

$$= 47.746 \times 10^3 \text{ N-mm}$$

$$F_t = \frac{2T_1}{D_1} = \frac{2 \times 47.746 \times 10^3}{5 \times 30}$$

$$= 636.6 \text{ N}$$

$$F_r = F_t \tan\phi$$

$$= 636.6 \tan 20^\circ$$

$$= 231.71 \text{ N}$$

Resultant force $(H = F_t + F_r)$

$= \sqrt{2}$ (H or V)

$= 1225.40$ N.

$P_1 = P_3$

$\Rightarrow T_1 \omega_1 = T_3 \omega_3$

$$\Rightarrow \frac{T_1}{T_3} = \frac{\omega_3}{\omega_1}$$

$$\frac{2(F_{t_1} \frac{D_1}{2})}{2(F_{t_2} \frac{D_3}{2})} = \frac{N_3}{N_1}$$

speed $\propto \frac{1}{Dia}$

$$\therefore \frac{F_{t_1} D_1}{F_{t_3} D_3} = \frac{D_1}{D_3}$$

P_1 & P_2
→ Powers

$$\Rightarrow \boxed{\frac{F_{t_1}}{F_{t_3}} = 1}$$

$$\therefore \boxed{F_{t_1} = F_{t_3}}$$

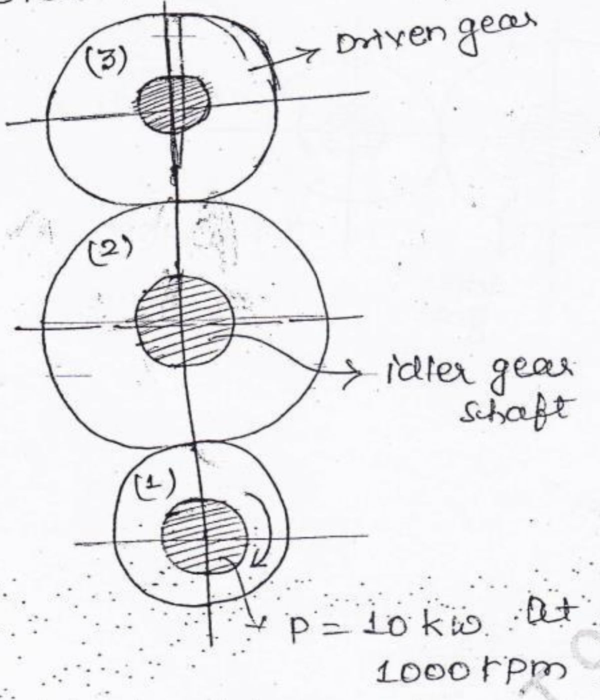
$F_{t_1} = F_{t_3}$ ($\because P_1 = P_3$)
$F_{r_1} = F_{r_3}$ ($\because \phi_1 = \phi_3$)
$F_{n_1} = F_{n_3} = F_n$

- Always F_r is in the dir^n of power transmission w.r.t gear.
- direction of F_r w.r.t driven gear will be in the dir^n of power transmission.
- F_t dir^n depends on the dir^n of rotation of that gear.
- F_r always along the line of centres of driver and driven gear, F_t is always along common tangent of the driver and driven gear.

(8) • Idler gear shafts are always designed based on Bending eq^n because it is not subjected to any T.M but power transmission shafts are always designed

based on Theories of failures because they are subjected to both B.M and T.M.

(8)



$m = 5mm$

$Z_1 = 20$

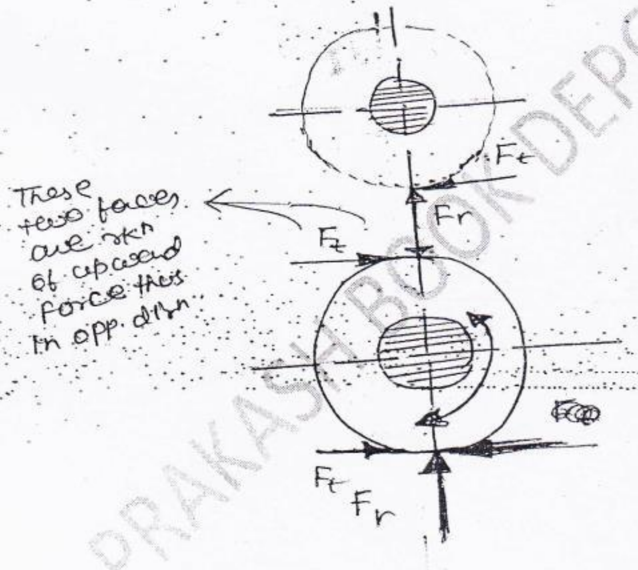
$Z_2 = 40$

$Z_3 = 60$

$\phi = 20°$

small → Driver
Bigger → Driven.

Here, power is transmitted from ~~top~~ bottom to top as rotation is shown only in the bottommost gear.
∴ F_r is dirn should be in same dirn of power transmission.
Then Draw tangent F_t in the dirn of rotation.



$H = 2F_t$

$V = 0$

$R = H = 2F_t$

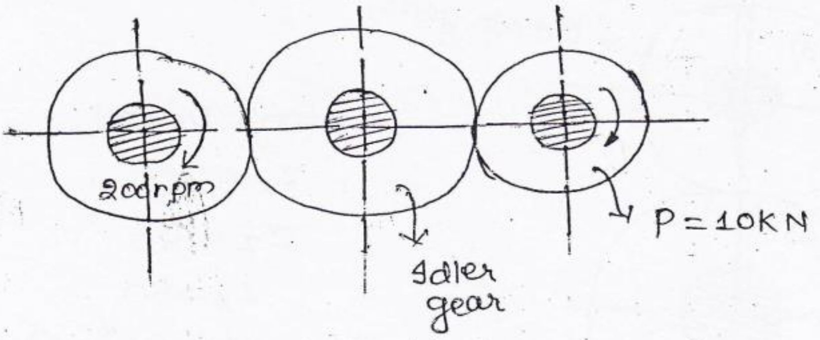
$$T_1 = \frac{P \times 60}{2\pi N_1} \times 10^6 = 9549 2.96 \text{ N-mm}$$

$$F_t = \frac{2T_1}{D_1} = \frac{2 \times 9549 2.96}{5 \times 20}$$

$$= 1909.85 \text{ N}.$$

$R = 2F_t = 3819.7N.$

(8)



$Z_1 = 15$

$Z_2 = 60$

$Z_3 = 30$

$m = 5mm$

$\phi = 20°.$

$$G = \frac{N_1}{N_2} = \frac{D_2}{D_1} = \frac{Z_2}{Z_1}.$$

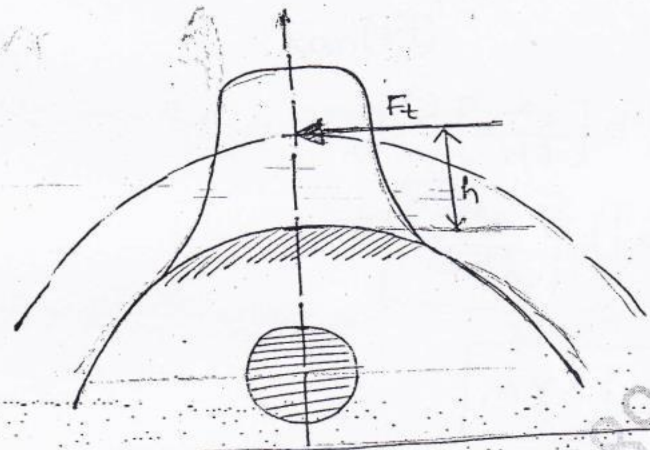
$$\frac{N_2}{N_3} = \frac{D_3}{D_2} = \frac{Z_3}{Z_2}$$

(or) $T_3 = \frac{P \times 60}{2\pi N_3}$

$$F_t = \frac{2T_1}{D_1} = \frac{2T_3}{D_3}$$

LEWIS EQUATION

Lewis beam strength equation

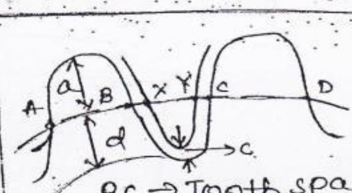


- clearance → C
  = (deddendum of gear) − (Addendum of gear pinion)
- Height above the pitch circle is called addendum.
- Below the pitch circle portion → flank.
- Above the pitch circle portion → face

a → Addendum,
d → Deddendum,

Backlash → BC − XY

XY → Tooth space of pinion



BC → Tooth space.

AB → Tooth thickness.

p_c = AC → pitch circle.

C → clearance.

condition for safe design of Gear tooth,

$$[(\sigma_b)_{max}]_{ind} \le (\sigma_b)_{per}$$

$$\frac{M_{max}}{Z_{N \cdot A}} \le [\sigma_b]$$

$$\frac{F_t \times h}{\frac{1}{6} b t^2} \le [\sigma_b]$$

$$\frac{6 F_t h}{\cancel{\frac{1}{6}} b t^2} \le [\sigma_b]$$

$$F_t \le \boxed{[\sigma_b] \times b \times \frac{t^2}{6h}} \rightarrow F_s$$

F_s = Beam strength of gear tooth

↳ Max^m value of F_t i.e, $(F_t)_{max}$.

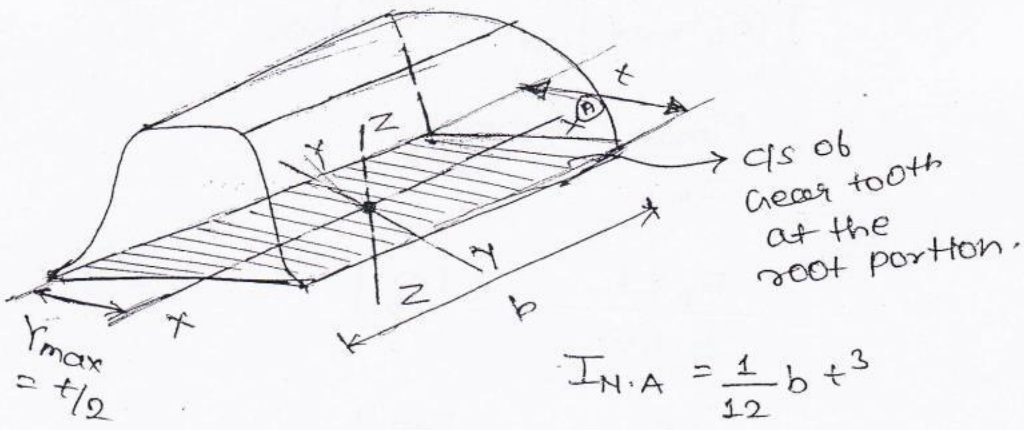
$$F_s = [\sigma_b]\, b \left[\frac{t^2}{6h}\right] \times \frac{m}{m}$$

$$F_{s'} = [\sigma_b]\, b \underbrace{\left(\frac{t^2}{6hm}\right)}_{\rightarrow Y} m$$

$$\boxed{F_s = [\sigma_b]\, b\, Y\, m}$$

$$\boxed{Y = \frac{t^2}{6hm} = \text{Lewis's form factor} = \pi y}$$

$\phi = 20° \Rightarrow y = 0.124 - \dfrac{0.512}{Z}$ (F.D)
$Y_2 > Y_1$ ($\because Z_2 > Z_1$)



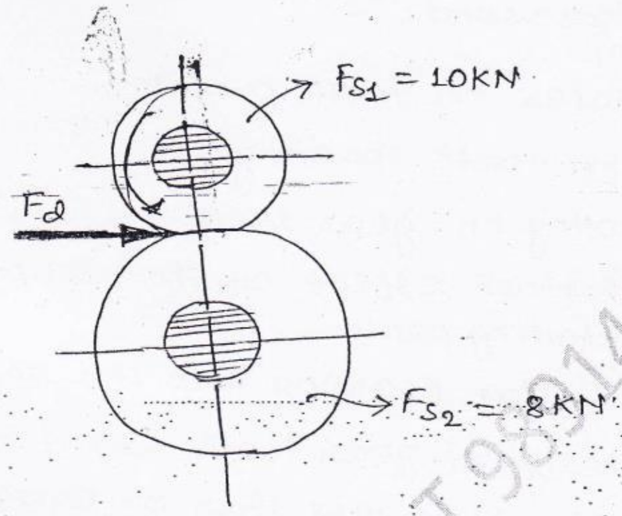
$$I_{N.A} = \frac{1}{12} b t^3$$

$$Y_{max} = t/2.$$

$$\boxed{Z_{N.A} = bt^2/6}$$

- weaker gear → least product of $\sigma_b Y$
- If both are made up of same material then pinion is weaker gear.
- Y is taken for the weaker gear.
- σ_b → Permissible bending stress.

(8)



$F_d \leq$ min. of $(F_{s_1}$ & $F_{s_2})$

F_d → Actual value of F_t acting on gear tooth of pinion and gear.

$F_d \leq$ Beam strength of weaker gear tooth of given gear pair

$F_d \leq$ min. of $(F_{s_1}$ & $F_{s_2})$

$$\boxed{F_d \leq \left([\sigma_b]Y\right)_{\text{weaker gear tooth}} b\, m}$$

Gear and pinion are made of same material,

$F_d \leq F_{s_1}$

$$\boxed{F_d \leq \left([\sigma_b]Y\right)_{\text{pinion}} b\, m}$$

Beam strength of a gear tooth represents maxm value of tangential load that a given gear tooth can withstand without bending failure. Hence, in any condition dynamic load acting on gear tooth should be less than beam strength of weaker gear.

Dynamic load

- It represents the actual value of tangential load (F_t) acting on the gear tooth in the dynamic condition.
- Dynamic load is always greater than F_t because of following reasons:-

(i) Inaccuracies in tooth profile.

(ii) Errors in tooth spacing.

(iii) Impact loads acting on gear tooth

(iv) Stress concentration effect at the root portion.

(v) Inertia of rotating parts.

(vi) Misalignment b/w bearing and the shaft.

For the safe design of gear tooth w.r.t bending failure F_d should be less than or equal to beam strength of weaker gear.

weaker gear is the gear which has minimum value for beam strength i.e, which has minm value of product of $[\sigma_b]$ and Y.

when gear and pinion are made of same material pinion is the weaker gear because it has minimum value of F_s

$$\left[\begin{array}{l} \because b_1 = b_2 \\ m_1 = m_2 \end{array} \right.$$

$$[\sigma_{b_1}] = [\sigma_{b_2}]$$

$$Y_1 = Y_2 (\because Z_1 < Z_2)$$

Approx method for the determination of actual dynamic load (F_d)

$$F_d = F_t \times C_v \quad (\text{or}) \quad \frac{F_t}{C_v}$$

$$C_v > 1 \qquad \boxed{C_v < 1}$$

$C_v \rightarrow$ velocity factor $= \frac{3+v}{3}$ (or) $\frac{3}{3+v}$ ($v \le 10$ m/s)

$$v = \frac{\pi D_1 N_1}{60} \text{ (or) } \frac{\pi D_2 N_2}{60} \text{ in m/s}$$

$$F_t = \frac{2T_1}{D_2} \text{ (or) } \frac{2T_2}{D_2}$$

Safe condition for gear tooth w.r.t bending.

$$F_d \le \left([\sigma_b]_Y\right)_{\text{weak gear}} b\,m$$

$$F_t \times C_v \le \left([\sigma_b]_Y\right)_{\text{weak gear}} b\,m$$

$$\boxed{\frac{2T_1}{D_1} \times C_v \le \left([\sigma_b]_Y\right)_{\text{weak gear}} b\,m}$$

$$\left([\sigma_b]_Y\right)_{\text{weak gear}} = \text{min. of} \left\{[\sigma_{b_1}]_{Y_1} \text{ \& } [\sigma_{b_2}]_{Y_2}\right\}$$

F_w = wear strength

$\hookrightarrow$ calculation of pinion only.

$$\boxed{F_w = D_1 Q K b}$$

$$\boxed{Q = \frac{2G}{G+1}}$$

'+' ⇒ Internal gears

'−' ⇒ External gears.

K = Material combination factor

$$\boxed{k = \frac{(\sigma_{es})^2 \sin\phi}{1.4}\left(\frac{1}{E_1}+\frac{1}{E_2}\right)}$$

$\sigma_{es} =$ Surface endurance limit (fatigue wear).

$F_d \leq F_w \Rightarrow$ safe w.r.t wear

$(F_d \leq F_s)$ $\quad F_d \leq F_s \Rightarrow$ safe w.r.t bending.

$$\boxed{F_w > F_s > F_d}$$

Safe condition for gear.

- Actual F_d is obtained by Buckingham Dynamic load eqn.

(Q) (13) $m = 3mm$, $z = 16$, $b = 36$, $\phi = 20°$, $P = 3kw$, $C_v = 1.5$,

If $C_v < 1$ (multiply otherwise divide).

$D = mz = 3 \times 16 = 54 mm.$

$$\frac{2T_1}{D_1} \times C_v \leq \left([\sigma_b]_Y\right)_{w.a} b \cdot m$$

$$T = \frac{3}{2\pi \times 20} \times 10^6 = 23.873 \times 10^3 \ N\text{-}mm.$$

$$\frac{2 \times 23.873 \times 10^3}{54} \leq .36 \times 0.3\,(\sigma_b) \times 3$$

$$\Rightarrow \sigma_b = 46.05 \text{ MPa}$$

If service factor is given then,

$$\boxed{K_s \times \frac{2T_1}{D_1} \times C_v \leq \left([\sigma_b]\, Y\right)_{\substack{weak\\gear}} b\, m}$$

Abrasive wear :- Due to some foreign particles in lubricant

Scoring :- Metal to metal contact in the absence of lubricant.

Corrosive wear :- Due to corrosive enviorment.

Pitting :- occur due to fatigue wear.

Design of welded joint

- 100% efficiency is possible
- cost of joint is less.
- 100% leak proof joint is possible.
- stress concentration is less.

Design of welded joint obtained by fusion welding process.
Welding is performed at high temp. thus
occurance residual stresses is common phenomena.
Due to this thermal distortion occurs.
Inspection of welded joint is time taking.
Strength and quality of joint in welded joint depend on labour's skill.

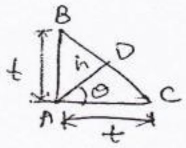
Types of welding

(1) Tack welds.
(2) Fillet welds.
(3) Butt welds.
(4) Edge welds

Tacking → joining of the plate temporarily as the plate position are not misaligned during welding.
Represented by • • • • •

Fillet weld

Represented by
(Right angle isosceles Δ)



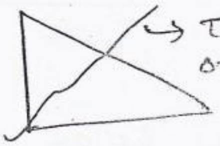
$t \rightarrow$ leg of weld.

$h \rightarrow$ throat of weld.

or, t = size of leg of fillet weld.

h = Throat thickness.

$$\boxed{h = \frac{t}{\cos\theta + \sin\theta}}$$



Throat thickness $\rightarrow$
Thickness of weld along failure plane.

- Fillet welds are specified by the size of leg of fillet weld (i.e, t)

$$\boxed{\text{Weld Area} = h \cdot l_e}$$

$\boxed{l_e = l}$ for single fillet weld joint.

$\boxed{l_e = 2l}$ for Double fillet weld joint.

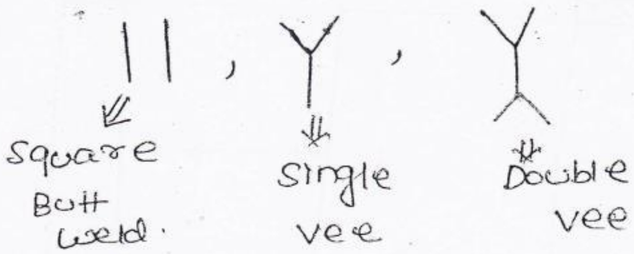
In parallel fillet weld, $\theta = 45^\circ$.

In transverse fillet weld, $\theta = 67\frac{1}{2}^\circ$.

$\theta \rightarrow$ location of throat plate.

<u>Butt weld</u>

It is represented by



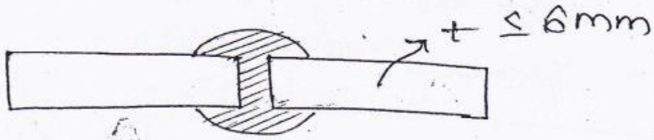
If $t < 6mm$ — Square

$6 < t < 20mm \Rightarrow$ Single V

$t > 20mm$ —— Double V

Edge weld:

Represented by semi-circular 



Square butt welded joint.

Types of fillet welds

(i) Parallel fillet welds.

(ii) Transverse fillet welds.

(iii) Compound fillet welds.

Parallel fillet welds

load is acting in a dirn which is parallel to the length of the welds.

Transverse fillet welds

loads acts in a dirn which is ⊥r to the length of welds.

Compound fillet welds

Combination of parallel and transverse fillet weld.

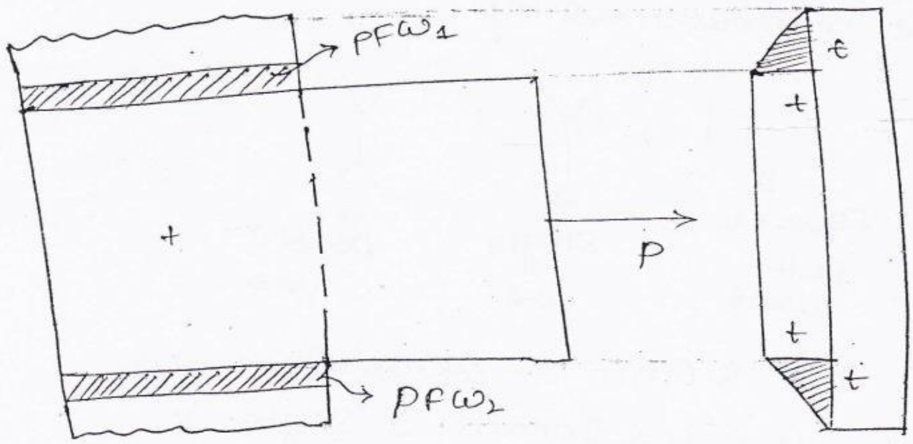


Fig:- Double parallel fillet welded lap joint

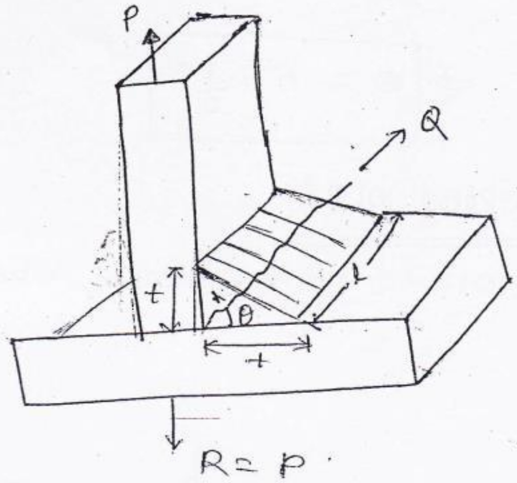


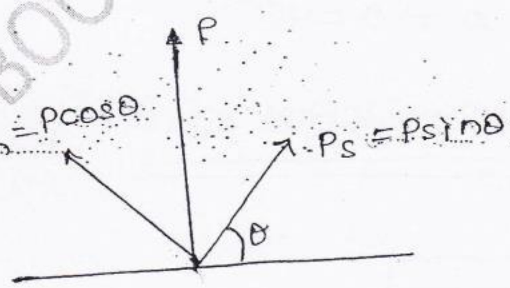
Fig:- Double transverse fillet welded T-joint

P → Inclined load w.r.t weld.

Q → shear load w.r.t weld.

w.r.t Q → the joint is double parallel fillet welded T-joint.

In transverse fillet weld



In case of transverse two stress developed shear and tensile but tensile stress is neglected so only shear stress is considered.

In tranverse

$\theta = 67.5°$

P_s = shear force $= P\sin\theta$

P_n = Tensile force $= P\cos\theta$

In design tensile force is neglected because $\cos 67.5°$ is less than $\sin 67.5°$.

$$\tau_s = \frac{P_s}{A_s} = \frac{P.\sin\theta}{h.l_e} = \frac{P\sin\theta}{\left(\frac{t}{\cos\theta + \sin\theta}\right).l_e}$$

To locate transverse plane,

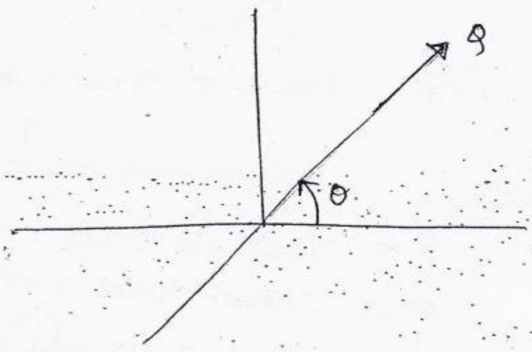
$$\frac{d(\tau_s)}{d\theta} = 0 \Rightarrow \boxed{\theta = 67\tfrac{1}{2}^\circ}$$

In parallel fillet welded joint

In parallel fillet joint, only shear load is present.

$$P_s = P$$

$$P_n = 0$$



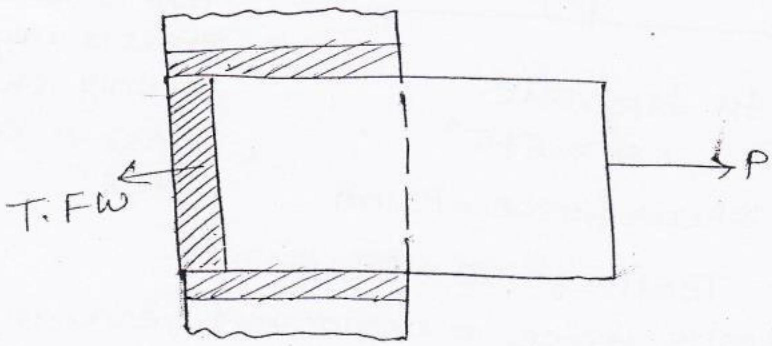
$$\tau_s = \frac{P_s}{A_s}$$

$$\tau_s = \frac{P_s}{t \cdot l_e}$$

$$\tau_s = \frac{P_\theta}{\left(\frac{t}{\cos\theta + \sin\theta}\right) \cdot l_e}$$

$$\frac{d\,\tau_s}{d\theta} = 0 \Rightarrow \theta = 45^\circ$$

Compound fillet welded joint



Parallel fillet welded joint	Transverse fillet welded joint
• Direction of load is parallel to the length of weld.	• Direction of load is perpendicular to the length of weld.
• Shear force = P.	• Shear force = Ps = $P \sin\theta$
• Tensile force = 0.	• Tensile force = $P \cos\phi$
• $\theta = 45°$	• $\theta = 67\frac{1}{2}°$
• $h = \frac{t}{\cos 45 + \sin 45°} = \frac{t}{\sqrt{2}} = 0.707t$	• $h = 0.765t$
• Area = $h \cdot le = 0.707 t \cdot le$	• Area = $h \cdot le = 0.765\, t\, le$
• Strength of weld $= 0.707\, t \cdot le \cdot \tau_{per}$	• Strength of weld $= 0.832\, t \cdot le \cdot \tau_{per}$.
$\tau_{ind} \le \tau_{per}$ $\frac{P_s}{0.707 tle} \le \tau_{per}$ $\frac{P}{0.707 tle} \le \tau_{per}$. $P \le 0.707\, t \cdot le \cdot \tau_{per}$.	$\tau_{ind} \le \tau_{per}$. $\frac{P_s}{0.707 t\, le} \le \tau_{per}$. $\frac{P \sin 67.5}{0.707\, t \cdot le} \le \tau_{per}$ $P \le 0.828\, t \cdot le \cdot \tau_{per}$.

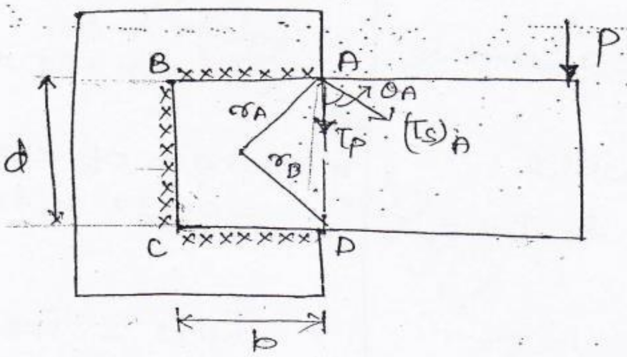
• For a given dimensions of the weld and given material, the strength of transverse fillet weld is 18% more than strength of parallel fillet weld transverse fillet weld is strongest weld [i.e, parallel fillet weld is the weakest weld.

- $$\frac{P_{\text{transverse fillet weld}}}{P_{\text{parallel fillet weld}}} = \frac{0.832}{0.707} = 1.176 = 1.18 = 18\%..$$

- In the design of fillet weld if unless or otherwise mentioned it is better to consider fillet weld as parallel fillet weld because it is weaker weld.

$$(\tau_{PFW} > \tau_{PFW})$$

Eccentric loading of welded joint



$$\tau_p = \frac{P}{A_w} = \frac{P}{0.707\, t\cdot l_e} \qquad [l_e = (2b+d)]$$

$$\tau_s \propto r$$

$$(\tau_s)_{max} = (\tau_s)_A = (\tau_s)_D = \frac{T \cdot r_{max}}{J_w}$$

$J_w \rightarrow$ Polar m.o.I of entire welding system.

$$(\theta_A = \theta_D) < (\theta_B = \theta_C)$$

$$(\tau_R)_{max} = (\tau_R)_A = (\tau_A)_D$$

$$\tau_R = \sqrt{\tau_p^2 + (\tau_s^2)_{max} + 2\,\tau_p\,(\tau_s)_{max}\cos\theta_{min}}$$

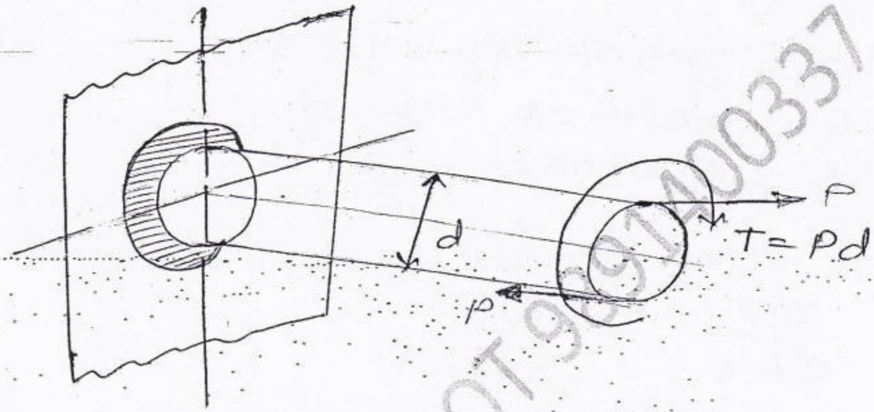
for safe design:-

$$(\tau_R)_{max} \leq \tau_{per}$$

$$\frac{z}{t} \leq \tau_{per}$$

$$t \geq \text{—} \ mm.$$

Circular fillet weld under pure torsion



Torsional shear stress,

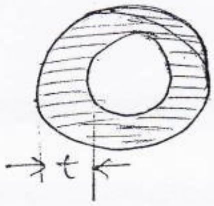
$$\tau_s = \frac{T}{Z_p} = \frac{Pd}{\frac{\pi d^2 h}{2}} = \frac{2Pd}{\pi d^2 h} = \frac{2T}{\pi d^2 \frac{t}{\sqrt{2}}}$$

$$\boxed{\tau_s = \frac{2.83\,T}{\pi d^2 t}}$$

Fillet weld under pure bending

$$\sigma_b = \frac{M}{Z_\omega} = \frac{M}{\frac{\pi}{4} d^2 h} = \frac{4M}{\pi d^2 h} = \frac{4\sqrt{2}\,M}{\pi d^2 t}$$

$$\boxed{\sigma_b = \frac{5.66 M}{\pi d^2 t}}$$



$$I = \frac{\pi d^3 t}{8}$$

$$J = \frac{\pi d^3 t}{4}$$

$$Z = \frac{\pi d^2 t}{4}, \quad Z_p = \frac{\pi d^2 t}{2}$$

Keys

$A_s = l \cdot b$, $A_c = \frac{l \cdot t}{2}$

$\tau_s = \frac{2T}{dbl}$, $\sigma_c = \frac{4T}{dt \cdot l}$

d = dia of shaft

$T = \frac{P}{\omega}$

l = length of key = $1.5d$

b = width of key = $\frac{d}{4}$

t = thickness = $\frac{d}{6}$

(Q) $(\tau_s)_{key} = (\tau_s)_{shaft}$

$$\frac{2T}{dbl} = \frac{16T}{\pi d^3}$$

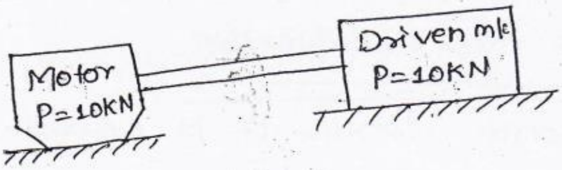
$$\Rightarrow \frac{l}{d^3} = \frac{\pi}{8b} = \frac{\pi}{8 \times \frac{d}{4}}$$

$$\Rightarrow \boxed{\frac{l}{d} = \frac{\pi}{2}}$$

20/02/14

Mechanical power transmission system (M.P.T.S)

Necessity of M.P.T.S

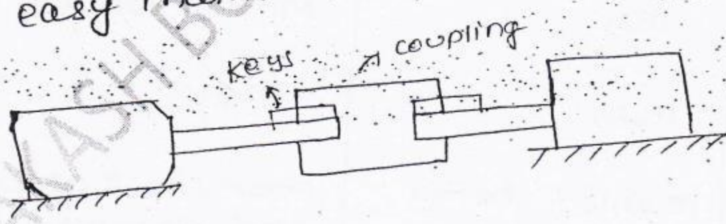


$P = T\omega = \text{const}$

$T \propto \frac{1}{\omega}$

Reason why driven and driver shaft mounted independently

(i) To obtain speed reduction.
(ii) To obtain variable speed at driven m/c.
(iii) To transmit power over longer centre distance
(iv) To drive more than one machine, by using a single prime mover.
(v) To obtain safety for driver and driven m/c in case of overloads.
(vi) for easy maintenance :-



Classification

MPTS
- Flexible P.T.S
  - Belt drives
    - Flat belt drive
    - V-belt drive
  - Chain drives
    - Bush & roller chains
    - Silent chains
  - Rope drives
    - Wire ropes
    - Fibre ropes.
- Non flexible P.T.S
  - Couplings
  - Clutches
  - Gear drives
    - Spur gear
    - Helical gear
    - Bevel gear
    - Worm gear.

Advantage of flexible over non-flexible

(i) cost is less.
(ii) used where there is larger centre-distance.
(iii) It has better damping capacity.

Disadvantages of flexible over non-flexible

(i) velocity ratio is not const. while it is const. in non-flexible.

(ii) Efficiency is less.

(iii) occupy more space.

Gear drives also called +ve drives i.e., $\frac{N_2}{N_1} = \text{const}$ i.e, velocity ratio is const.

Parameter	Flexible P.T.s	Non-flexible PTS
(i) centre dist.	larger C.D.	Smaller C.D
(ii) velocity ratio.	not const.	const.
(iii) Efficiency	less	more.
(iv) cost.	less	More costlier.
(v) service life	less	more
(vi) Sock absorption & damping capacity.	exhibit better capacity	poor shock absorption and damping capacity.

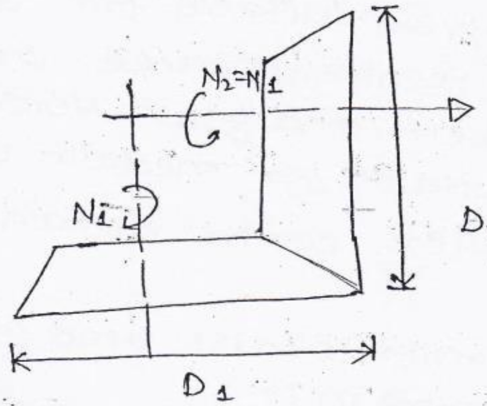
* Rope drives is suitable for long centre distance about 150 m.
* wire ropes about centre dist. of 60 m.
* Fibre ropes of 15 m.
* wire ropes exhibit high strength to weight

ratio so suitable for long distance.

* v-belts drives suitable for smaller centre distance,
* flat belt drives suitable for medium centre distance.
* chain drives suitable for smaller as well as longer centre distance. chain drives are in b/w belt and gear. Noise is more in chain drives. suitable for medium centre dist.
* for smaller centre distance gears ~~are~~ best.
* Bush and roller chains used in automobiles. Noise is more in it.
* silent chains are used where noise is constrained.
* spur gears and helical gears suitable for parallel shafts which are at smaller centre dist.
* spur gears are not suitable for high speed.
* Drawback of helical gear is that it is costly.
* spur gear suitable for low and medium pitch line velocities.
* Helical gears used for high-pitch line velocities
* Bevel gears used for non-parallel and intersecting shaft.
* worm gears used where speed reduction is required.

* when the driver shaft and Driven shaft are rotated on same r.p.m but in opposite dirn then as we use mitre gears.

Automobile Differential



Mitre gears

Two equal size bevel gears which are used to transmit power b/w two right angled inter-secting shafts rotating at same r.p.m but in opp. dir^n.

Types of flat belt drives

(1) open belt Drive → Similar to Internal gears
(2) cross belt Drive → " " External "
(3) Compound Belt drive
~~(4) Fast Belt Drive~~
(4) Fast and loose pulley drive
(5) stepped pulley drive.

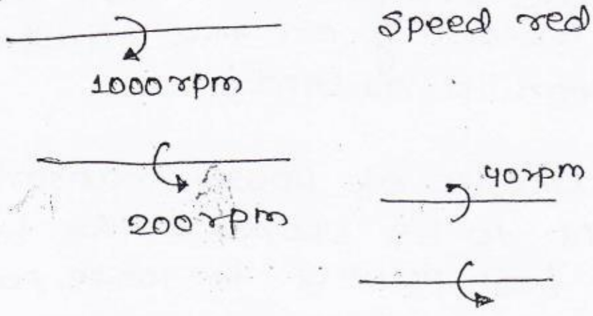
used to transmit power b/w two parallel shafts which are at medium centre Distance.

(6) Quater turn Drive (or) Right angled drive

Flat belt → Rectangular c/s.

V-belt → Trapezoidal c/s.

Compound belt drive is similar to gear train



Speed red^n = $\frac{1000}{200}$ = 5.

= speed red^n = 5.

speed red^n = 4.

Compound Belt Drive

To obtain higher speed red^n b/w driver & driven consist of multiple open belt drive and cross belt drive.

Fast and loose pulley drive is similar to clutche used to transmit power b/w to two shaft when intermittent service is required. (frequent stopping & starting are required).

Stepped pulley → Different steps are used.

→ open belt drive

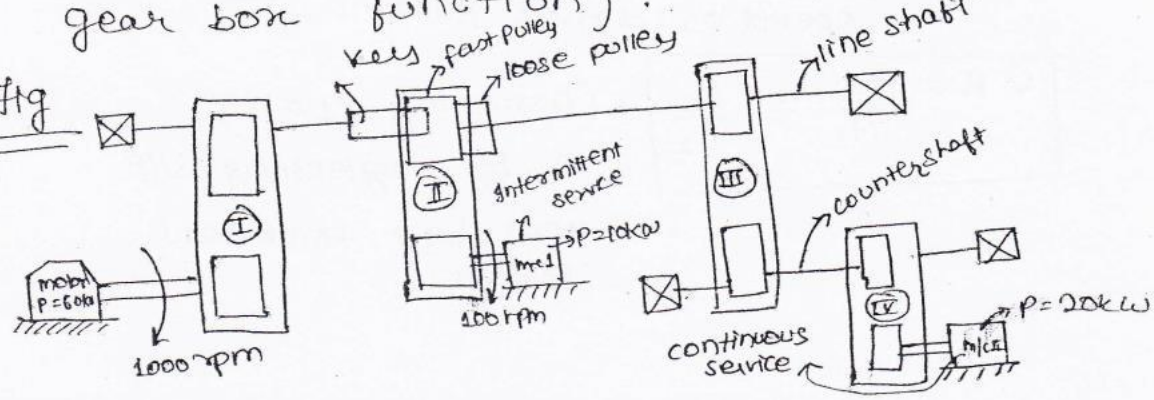
used when driver and driven shaft are rotating in same dir^n.

→ cross belt drive

used when driver and driven shaft are to rotate in opp. dir^n.

→ Stepped pulley is used to obtain variable speeds at driven shaft (similar to gear box function).

Fig



→ fast pulley is a pulley which is having key connection with the shaft whereas loose pulley is freely rotating on the shaft (∵ key connection is absent).

→ fast pulley is capable of power transmission.

→ whenever m/c is to be stopped the belt is shifted from fast pulley to loose pulley.

→ This drive f^n is similar to ~~function~~ of clutch.

This drive is used in the application where driven m/c requires intermittent service (frequent stoppings and startings are required)

→ Counter shaft used for higher speed reduction.

$$P = T\omega$$

$$P = (T_1 - T_2)\, v$$

$$\Rightarrow \quad T_1 - T_2 = \_\_\_\_$$

$$\frac{T_1}{T_2} = \_\_\_\_$$

$$T_1 = T_{max} = \sigma_{per} \times b \times t \quad \text{---} (1)$$

from eq(1) (b) width can be calculated

t (thickness) is determined by bending stress eqn.

Velocity Ratio

$$V.R = \frac{\text{speed of driven pulley}}{\text{speed of driver pulley}} = \frac{N_2}{N_1}$$

$$\boxed{V.R = \frac{N_2}{N_1} = \frac{D_1}{D_2}} \Rightarrow \text{(Assuming } v_1 = v_2)$$

i.e, by neglecting slip and belt thickness effect.

$$\boxed{V.R = \frac{N_2}{N_1} = \left(\frac{D_1+t}{D_2+t}\right)\left(1-\frac{s}{100}\right)}$$ ⇒ considering slip and thickness.

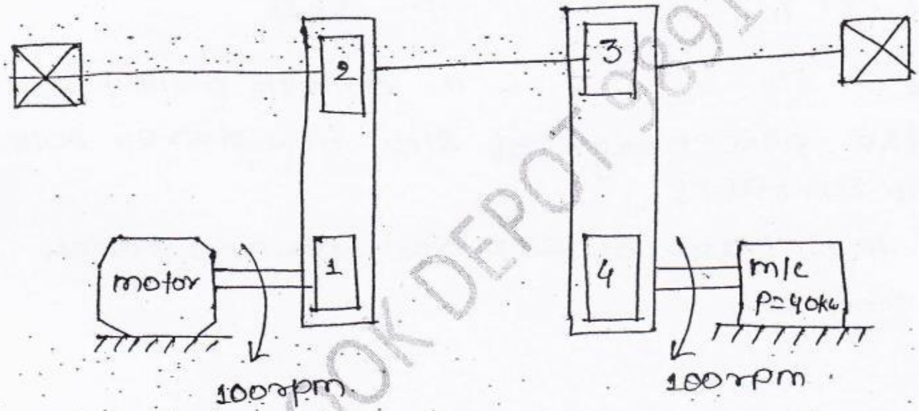
s = % of total slip.

$s = s_1 + s_2$.

s_1 = % of slip b/w driver pulley and the belt.

s_2 = % of slip b/w belt and driven pulley.

Compound Belt Drive



velocity ratio

$$\boxed{V.R = \frac{N_n}{N_1} = \left(\frac{D_1+t}{D_2+t}\right)\times\left(\frac{D_3+t}{D_4+t}\right)\times \cdots \left(\frac{D_{n-1}+t}{D_n+t}\right)\left(1-\frac{s}{100}\right)}$$

$$s_1 = s_1 + s_2 + s_3 + s_4 + \cdots + s_n$$

Here,

$$\boxed{V.R = \left(\frac{D_1+t}{D_2+t}\right)\times\left(\frac{D_3+t}{D_4+t}\right)\left[1-\frac{s}{100}\right)}$$

$$\frac{N_2}{N_1} = \frac{D_1}{D_2} \quad \text{—— (I)} \qquad \frac{N_4}{N_3} = \frac{D_3}{D_4} \quad \text{—(II)}$$

$$\frac{N_4}{N_1} = \frac{N_2}{N_1} \times \frac{N_4}{N_3} \quad (\because N_2 = N_3)$$

$$\frac{N_4}{N_1} = \frac{D_1}{D_2} \times \frac{D_3}{D_4}$$

Effect of slip on velocity ratio

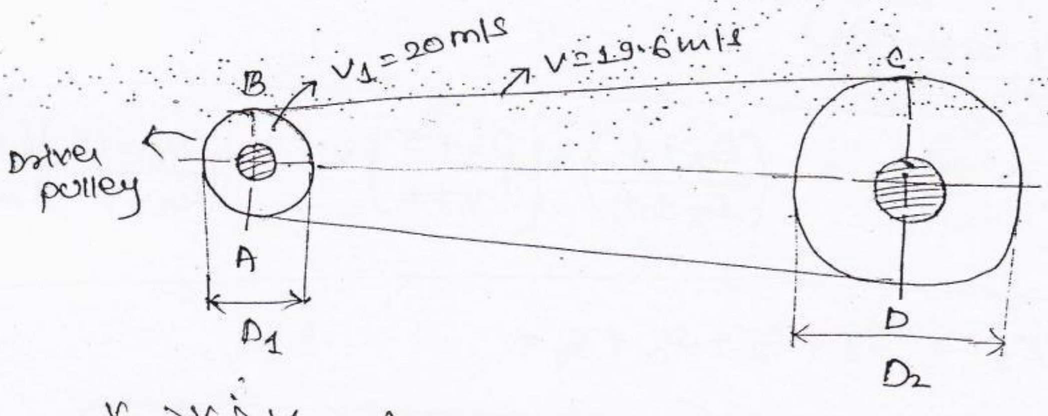
Let V_1 = linear velocity of driver pulley.

V = " " " belt.

V_2 = " " " driven pulley.

S_1 is the percentage of slip b/w driver pulley and belt surfaces

S_2 is the %age of slip b/w belt and driven pulley surfaces.



$V_1 > V > V_2$ due to lack of friction due to insufficient friction grip (belt is moving slower than driver pulley), insufficient grip occurs because when belt is entering it carries some amount of air with it, so in flexible, slip can't be avoided due to air layer.

$$\boxed{V_1 = \frac{\pi D_1 N_1}{60}}$$

$$\boxed{v = V_1 - V_1 \times \frac{S_1}{100} = V_1\left[1 - \frac{S_1}{100}\right]}$$

$V_1 = 196\ m/s$

slip $= 0.4\ m/s$, % slip $= 2\%$.

$$V_2 = v - \frac{v \times S_2}{100} = v\left[1 - \frac{S_2}{100}\right]$$

$$\boxed{V_2 = v\left(1 - \frac{S_2}{100}\right)} \quad \text{— (iii)}$$

From eqn (ii) & (iii)

$$V_2 = V_1\left(1 - \frac{S_1}{100}\right)\left(1 - \frac{S_2}{100}\right)$$

$$\frac{\pi D_2 N_2}{60} = \frac{\pi D_1 N_1}{60}\left[1 - \left(\frac{S_1 + S_2}{100}\right) + \underbrace{\frac{S_1 S_2}{100}}_{\text{Neglected}}\right]$$

$$\boxed{\frac{N_2}{N_1} = \frac{D_1}{D_2}\left[1 - \frac{S}{100}\right]} \quad \text{where } S = S_1 + S_2$$

Power output $= T_2 \omega_2$ $\qquad P_o < P_i$

Input power $= T_1 \omega_1$

$$\boxed{\eta = \frac{P_o}{P_i} = \frac{T_o \omega_o}{T_i \omega_i}}$$

$$\eta = \frac{(T_1 - T_2) R_2 \omega_2}{(T_1 - T_2) R_1 \omega_1} = \frac{V_2}{V_1}$$

$$\boxed{\eta = \frac{V_2}{V_1}}$$

* Slip is defined as the relative motion b/w belt and pulley surfaces.
* Relative motion exists b/w belt and pulley surfaces due to insufficient frictional grip (i.e, due to the existence of air layer) b/w belt and pulley surfaces.
* In presence of slip belt moves slower than driver pulley velocity but faster than driven pulley velocity i.e,

$$V_1 > V > V_2$$

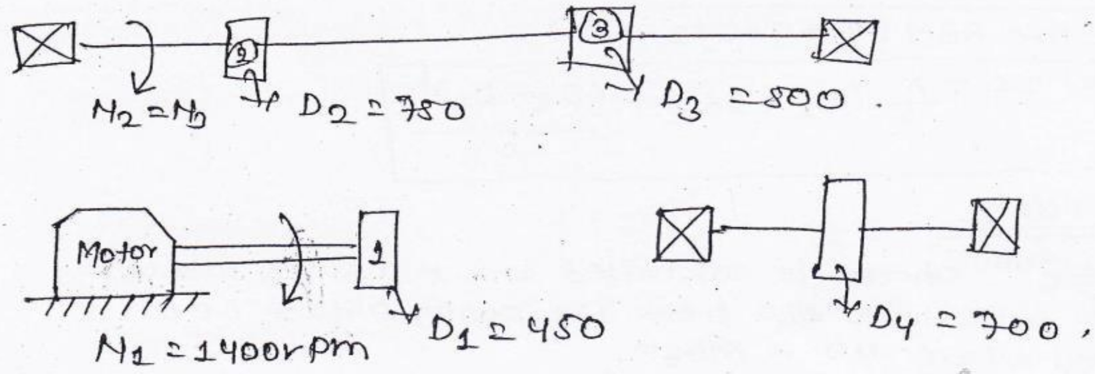
* The effect of slip is to decrease the speed of driven pulley, velocity ratio and efficiency of the belt drive.

(9) A motor drives a main shaft by means of flat belt, the dia. of the pulleys on the motor shaft and main shaft are 450mm and 750mm resp. Another pulley of dia. 500mm drives a counter shaft having a pulley of dia 700mm. The pulley of dia 500mm is mounted on main shaft. If the slip at each drive is 3%. Calculate the speed of counter shaft if motor runs at 1400 rpm.

Sol:→

$$\frac{N_2}{N_1} = \frac{D_1}{D_2}\left[1 - \frac{s}{100}\right]$$

$$\Rightarrow \frac{N_2}{1400} = \frac{450}{750} \times 0.94.$$



$$\frac{N_4}{N_1} = \frac{D_1 \times D_3}{D_2 \times D_4}\left(1 - \frac{s}{100}\right)$$

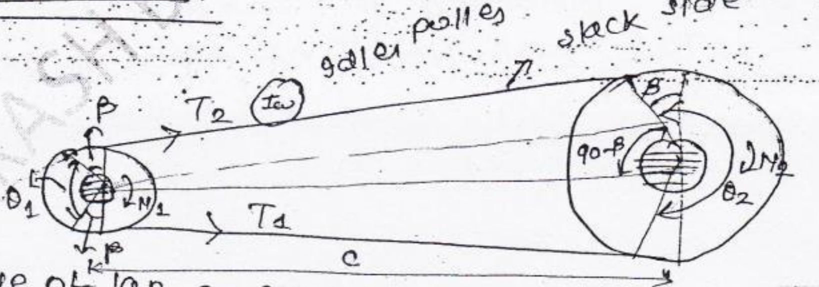
$$\Rightarrow \frac{N_4}{N_1} = \frac{450 \times 500}{700 \times 750}\left[1 - \frac{\overset{3\%}{(S_1+S_2)} + \overset{3\%}{(S_3+S_4)}}{100}\right]$$

$$s = s_1 + s_2 + s_3 + s_4 = 6\%.$$

$$\frac{N_4}{1400} = \frac{450 \times 500}{700 \times 750}\left(1 - \frac{6}{100}\right)$$

$$N_4 = 564\ rpm$$

<u>open belt drive</u>

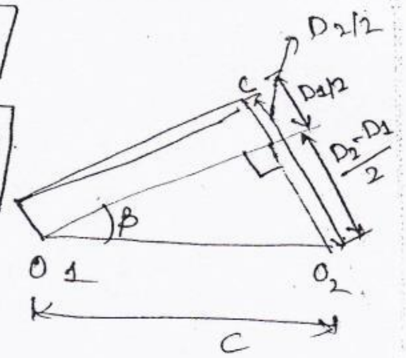


θ_1 = angle of lap or contact or wrap at the driver pulley
θ_2 = " " " " " " " " " driven ".

$$\boxed{\begin{array}{l}\theta_1 = \pi - 2\beta \\ \theta_2 = \pi + 2\beta\end{array}} \qquad \boxed{\theta_1 + \theta_2 = 2\pi \text{ radians}}$$

$$\boxed{\beta = \sin^{-1}\left(\frac{D_2 - D_1}{2C}\right) \times \frac{\pi}{180} \text{ radians}}$$

$$\boxed{\sin\beta = \frac{O_2 D}{O_1 O_2} = \frac{D_2 - D_1}{2C}}$$



Length of open belt drive

$L = \text{Arc } AB + BC + \text{arc } CD + AD$

$$\boxed{L = 2C + \frac{\pi}{2}(D_1 + D_2) + \frac{(D_2 - D_1)^2}{4C}}$$

Imp. points

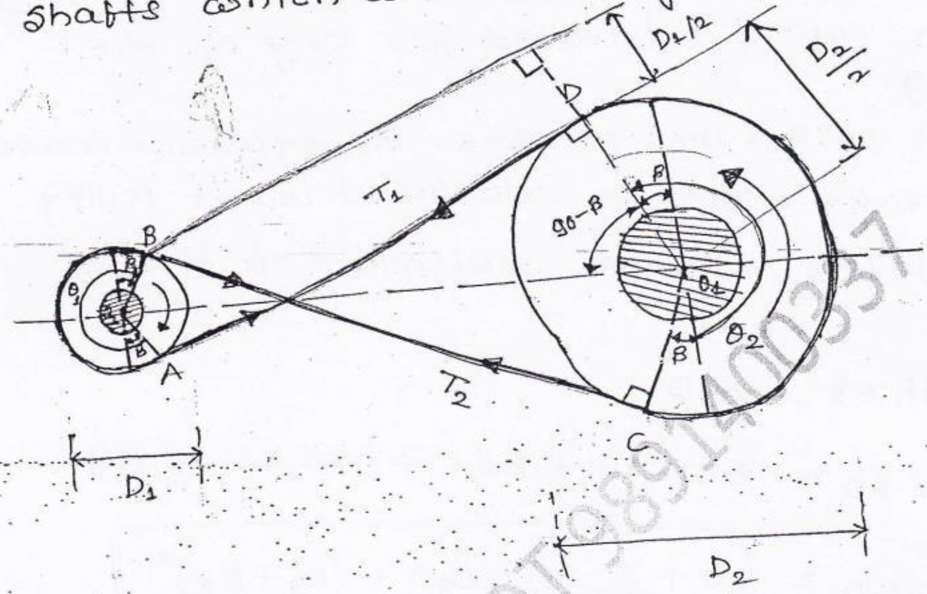
- $\frac{T_1}{T_2} = e^{\mu\theta}$ should be calculated w.r.t a pulley where the belt is likely to slip from the pulley surface i.e at a pulley where $\mu\theta$ is minm.
- ~~In open belt drive when both the pulleys are made up of~~ same material the belt is likely to slip from smaller pulley because here $\mu\theta$ is minm.
- If $C\uparrow$ $\beta\uparrow$ when $\beta\uparrow$ $\theta_1\downarrow$ & $\theta_2\uparrow$. $\theta_1\downarrow$ means less contact b/w pulley and belts and hence $\mu\theta$ ↓ so the belt will slip out easily from smaller pulley - so this type of belt is not suitable for short centre distance. Hence in open belt drive $\boxed{C \geq C_{min}}$
- When centre distance $C < C_{min}$, alternative for an open belt drive are :-
  - (i) V-belt drive.
  - (ii) open belt drive with an idler pulley.

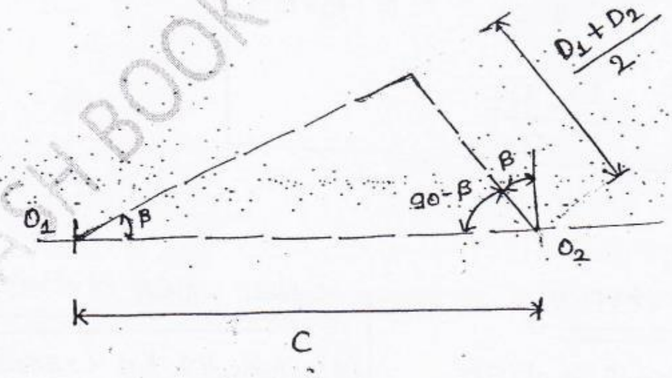
Idler pulley mounted on slack side, and acts as TSL. The function of idler pulley to increase the angle of contact when centre dist. is minm.

21 | 02 | 14

CROSS BELT DRIVE (C.B.D)

→ is used to transmit power b/w two parallel shafts which are rotating in opp. dirn.





$$\sin\beta = \frac{O_2E}{O_1O_2} = \frac{D_1+D_2}{2C}$$

$$\theta_1 = \theta_2 = \pi + 2\beta$$

where $\beta = \sin^{-1}\left(\frac{D_2+D_1}{2}\right) \times \frac{\pi}{180}$ radians.

$$C \downarrow \Rightarrow \beta \uparrow \Longrightarrow (\theta_1 \;\&\; \theta_2)\uparrow$$

when both drive are made up of same material then T_1/T_2 (or) T_2/T_1 both are same.

- When both the pulleys are made up of same material belt will slip out from the pulleys. Hence, $\frac{T_1}{T_2}$ ratio can be calculated for any of the pulley.
- When pulleys are made up of different materials $\frac{T_1}{T_2}$ ratio should be calculated w.r.t pulley which has minimum coefficient of friction.

- Length of C.B.D

$$L_{C.B.D} = \text{Arc } AB + BC + \text{Arc } CD + AD$$

$$\boxed{L_{C.B.D} = 2c + \frac{\pi}{2}(D_1 + D_2) + \frac{(D_2 + D_1)^2}{4c}}$$

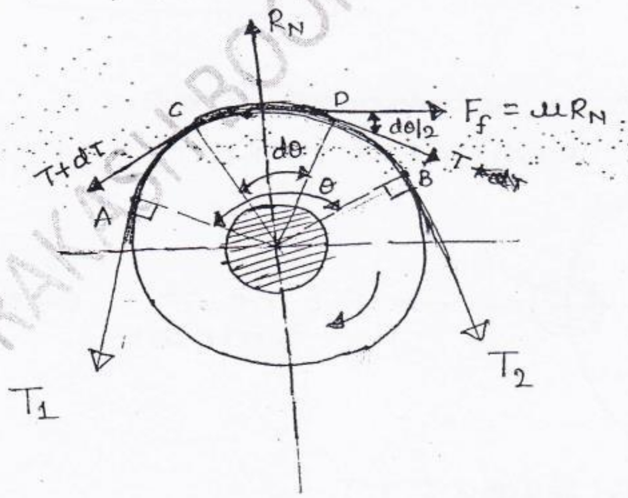
$$\boxed{\begin{aligned} \Delta L &= L_{C.B.D} - L_{O.B.D} \\ &= \frac{D_1 D_2}{2} \end{aligned}}$$

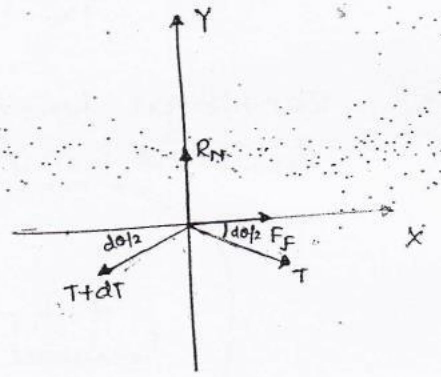
Comparision

Open Belt drive	Cross - Belt drive
(1) should be used when Driver and Driven pulley rotate in same dirn.	(1) should be used when Driver and Driven pulley rotate in opposite dirn.
(2) $\theta_1 = \pi - 2\beta$ $\theta_2 = \pi + 2\beta$ $\theta_1 + \theta_2 = 2\pi$	(2) $\theta_1 = \theta_2 = \pi + 2\beta$.
(3) $\beta = \sin^{-1}\left(\frac{D_2 - D_1}{2C}\right) \times \frac{\pi}{180}$	(3) $\beta = \sin^{-1}\left(\frac{D_2 + D_1}{2C}\right) \times \frac{\pi}{180}$

(4) $\frac{T_1}{T_2}$ ratio is less.	(4) $\frac{T_1}{T_2}$ ratio is more.
(5) centre distance $\boxed{C \geq C_{min}}$ (min^m centre distance)	(5) No constraint on centre Distance.
(6) when $\boxed{C < C_{min}}$, idler pulley is required.	(6) Idler pulley are not required.
(7) cost is less.	(7) cost is more.
(8) service life is more.	(8) service life is less.
(9) $L_{OBD} = 2C + \frac{\pi}{2}(D_1 + D_2) + \frac{(D_2 - D_1)^2}{4C}$	(9) $L = 2C + \frac{\pi}{2}(D_1 + D_2) + \frac{(D_2 + D_1)^2}{4C}$

$\frac{T_1}{T_2}$ ratio





$$\Sigma H = 0 \Rightarrow \mu R_N + T\cos\frac{d\theta}{2} - (T+dT)\cos\frac{d\theta}{2} = 0 \quad \text{---} (1)$$

$$\Sigma V = 0 \Rightarrow R_N - (T+dT)\sin\frac{d\theta}{2} - T\sin\frac{d\theta}{2} = 0 \quad \text{---} (2)$$

for smaller angles,

$$\sin\frac{d\theta}{2} \simeq \frac{d\theta}{2}$$

$$\cos\frac{d\theta}{2} \simeq 1$$

From eq^n (1)

$$\mu R_N + T - T - dT = 0$$

$$\boxed{\mu R_N = dT} \rightarrow (3)$$

from eq^n (2),

$$R_N - T\frac{d\theta}{2} - dT\frac{d\theta}{2} - T\frac{d\theta}{2} = 0$$

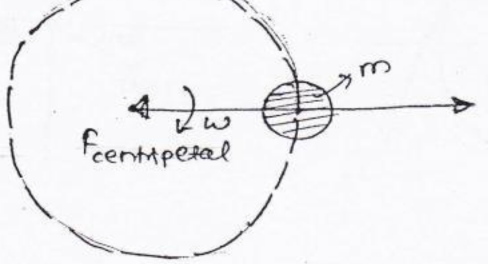
$$\boxed{R_N = T d\theta} \rightarrow (4)$$

$$\mu(T d\theta) = dT$$

$$\int_{T_2}^{T_1} \frac{dT}{T} = \int_0^{\theta} \mu \, d\theta$$

$$\log_e \frac{T_1}{T_2} = \mu\theta \Rightarrow \boxed{\frac{T_1}{T_2} = e^{(\mu\theta)_{min}}}$$

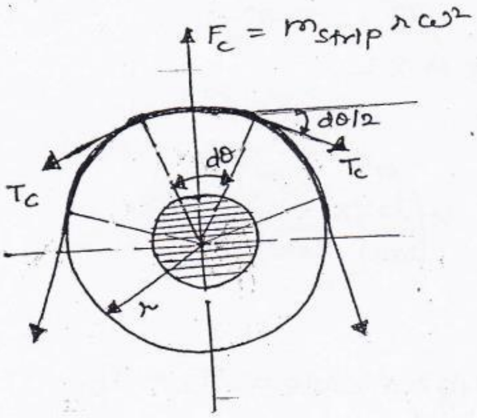
(F_c) centrifugal force → unbalanced force
Balanced using centripetal force.

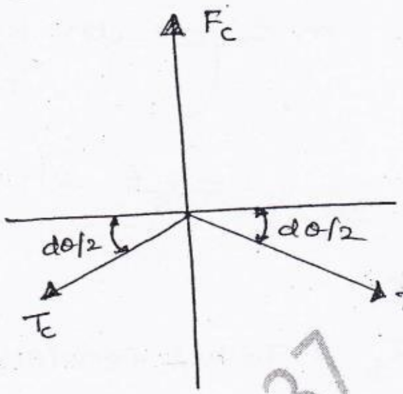


$F_c = me\omega^2$ e → eccentricity
$= mr\omega^2$

Expression for centrifugal tension (T_c) :-

centrifugal tension is the additional tension developed in the belt in presence of centrifugal force.





Let m = mass of the belt per unit length of 1 m

$= kg/m$

$m_{strip} = m r d\theta$ in kg

$F_c = (m r d\theta) r\omega^2 = m r^2 \omega^2 d\theta$

$= m v^2 d\theta$ in N.

$\Sigma V = 0$

$F_c - T_c \sin\frac{d\theta}{2} - T_c \sin\frac{d\theta}{2} = 0$

$F_c = 2T_c \sin\frac{d\theta}{2}$

$F_c = 2T_c \left(\frac{d\theta}{2}\right) \quad \left[\because \sin\frac{d\theta}{2} \simeq \frac{d\theta}{2}\right]$

$F_c = T_c d\theta$

$mv^2 d\theta = T_c d\theta$

$\boxed{T_c = mv^2}$

kg/m (Mass per unit length)

$\boxed{mv^2 = \frac{kg}{m} \times \frac{m^2}{sec} = kg\,m/sec = Newton}$

$m = \rho \times V$

$m = \left(\rho \text{ in } \frac{kg}{m^3}\right) \times \overset{m^2}{A} \times \overset{1m}{L}$

$= \rho \text{ in } \frac{kg}{m^3} \times \left(\frac{\overset{mm}{b}}{1000}\right)_m \times \left(\frac{\overset{mm}{t}}{1000}\right)_m \times 1\,m$

Let

T_t = Total tension on tight side $= T_1 + T_c$

In questions

[If mass density is not given then velocity is less and effect of T_c can be neglected]

$v < 8\ m/sec \Rightarrow$ Effect of T_c can be neglected.

$v \geq 8\ m/sec \Rightarrow$ Effect of T_c should be considered.

T_s = Total tension on slack side $= T_2 + T_c$

T_{max} :-

Condition for safe design of belt :-

$$\left[(\sigma_t)_{max}\right]_{ind} \leq \sigma_{per}$$

$$\frac{T_t}{A} \leq \sigma_{per}$$

$$T_t \leq A\, \sigma_{per}$$

$$T_t \leq b \times t \times \sigma_{per} \rightarrow T_{max} = 10\ KN.$$

Meaning :- At any condition the tension on the tight side should not exceed T_{max}.

$$\boxed{T_{max} = b \times t \times \sigma_{per}}$$

$$T_t = T_1 + T_c = T_{max}$$

$$\boxed{T_1 = T_{max} - T_c}$$

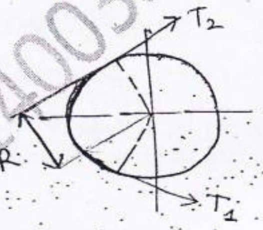
if $T_c = 0$, $\Rightarrow T_1 = T_{max}$

(P.T.C) Power transmission capacity of a Belt Drive

$$P = T\omega$$

$$P = (T_1 - T_2) R\omega$$

$$\boxed{P = (T_1 - T_2)\, V}$$



P.T.C can be ↑ by ↑ T_1

Belt drives are not suitable for higher velocities

Effect of centrifugal tension on P.T.C

$$P = (T_1 - T_2)\, v$$

$$P = T_1 \left[1 - \frac{T_2}{T_1}\right] v$$

$$P = T_1 \left[1 - \frac{1}{(T_1/T_2)}\right] v$$

$$P = T_1 \left[1 - \frac{1}{e^{\mu\theta}}\right] v$$

$$\boxed{P = T_1 \left[1 - \frac{1}{K}\right] v} \quad (K = e^{\mu\theta})$$

$$\boxed{P = T_1 K^1 V} \quad \left(\begin{array}{l}\text{where,} \\ K' = \left(1 - \frac{1}{K}\right)\end{array}\right)$$

when centrifugal Tension,

$$T_c = 0 \Rightarrow T_1 = T_{max}$$

$$\boxed{P = T_{max} k' V} \Rightarrow (II)$$

If $T_c \neq 0 \Rightarrow T_1 = T_{max} - T_c$

$$\boxed{P = (T_{max} - T_c) k' V} \text{ — (III)}$$

From eq^ns (II) & (III), we can conclude that P.T.C of a Belt drive decreases in presence of centrifugal tension. Hence, it is harmful w.r.t P.T.C w.r.t belt drive.

<u>condition for max^m power transmission</u>

$$P = T_1 k' V \longrightarrow (1)$$

$$P = (T_{max} - T_c) k' V$$

$$P = T_{max} k' V - k' m V^3$$

$$\frac{dP}{dV} = 0 \Rightarrow T_{max} k' - k' (3mV^2) = 0$$

$$\Rightarrow \cancel{T_{max} k'} \; (\cancel{1 - 3mV^2}) = 0$$

$$\Rightarrow k' (T_{max} - 3mV^2) = 0$$

$$\Rightarrow k' \neq 0, \quad T_{max} - 3mV^2 = 0$$

$$\Rightarrow \boxed{T_{max} = 3mV^2 \text{ (or) } 3T_c}$$

(1) $T_{max} = 3T_c$ (or) $\boxed{T_c = T_{max}/3}$

(2) $T_1 = T_{max} - T_c = 2T_c$

(3) $T_{max} = 3mV^2_{max}$ (or) $\boxed{V_{max} = \sqrt{\frac{T_{max}}{3m}}}$

Steps used in the determination of max^m P.T.C of a Belt drive

(1) $T_{max} = \sigma_{per} \times b \times t = $ ______ N.   ($\sigma_{per} \Rightarrow$ MPa, $b \Rightarrow$ mm, $t \Rightarrow$ mm)

(2) $T_c = \frac{T_{max}}{3} = $ ______ N.

(3) $T_1 = 2T_c = $ ______ N.

(4) $\frac{T_1}{T_2} = e^{(\mu\theta)_{min}} \Rightarrow T_2 = $ ______ N.   ($\theta \Rightarrow$ radians)

(5) $m = (\rho \text{ in } kg/m^3)\left(\frac{b}{1000}\right)\left(\frac{t}{1000}\right) \times 1\,m$

$= $ ______ kg/m.

(6) $v_{max} = \sqrt{\frac{T_{max}}{3m}} = $ ______ m/sec.

(7) $P_{max} = (T_1 - T_2)\,v_{max} = $ ______ watts.

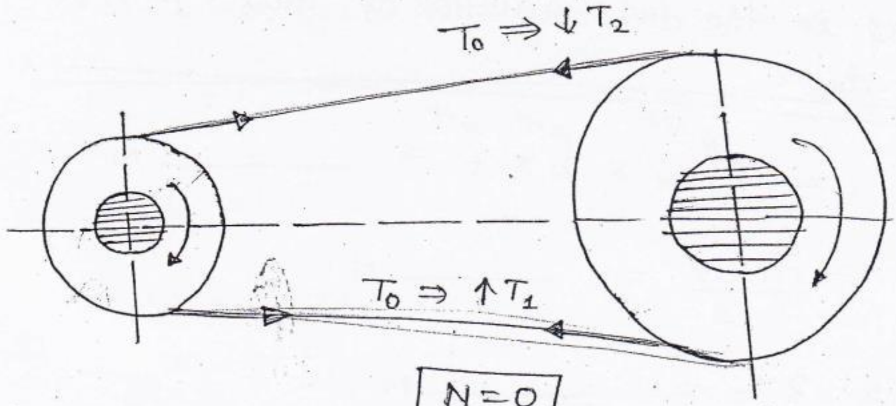
Initial tension

First tension developed in belt before T_1 & T_2, when belt is in stationary position.

Initial tension is the tension developed in the belt when it is in the stationary condition.

Initial tension is provided in the belt by taking a length of belt less than the required length of the belt.

In presence of initial tension the power transmission capacity of belt drive increases. Hence, it is useful w.r.t P.T.C of a belt drive.



$\boxed{N=0}$

$L \downarrow \Rightarrow T_0 \uparrow$

$\Rightarrow F_f \uparrow$

$\Rightarrow T_1 \uparrow$ & $T_2 \downarrow$

Increase in tension on tight side $= (T_1 - T_0)$ in N

Decrease in tension on slack side $= (T_0 - T_2)$ in N.

Let α = change in length of the belt per unit force of 1 N in m/N.

Increase in length of belt on tight side $= (T_1 - T_0)\alpha$ in m.

Decrease in length of the belt on slack side $= (T_0 - T_2)\alpha$ in m.

(Increase in length of the belt) = (Decrease in length of the belt)

$$(T_1 - T_0)\alpha = (T_0 - T_2)\alpha$$

$$\boxed{T_0 = \frac{T_1 + T_2}{2}}$$

If centrifugal tension is also considered

$$\boxed{T_0 = \frac{T_1 + T_2 + 2T_c}{2}}$$

Increase in tension on tight side $= (T_1 + T_c - T_0)$ N.

Decrease in tension on slack side $= (T_0 - T_2 - T_c)$ N

Let α = change in length of the belt per unit force of 1N in m/N.

Increase in length of belt on tight side

$= (T_1 + T_c - T_0)\alpha$ in m

Decrease in length of belt on slack side

$= (T_0 - T_2 - T_c)\alpha$ in m

Different methods to increase P.T.C

(1) $\mu \uparrow$ (2) $\theta \uparrow$ (3) $T_0 \uparrow$ (Best method among three for $\uparrow$ P.T.C)

INPUT DATA

(1) N_1 = ______ rpm ; N_2 = ________ rpm

(or) velocity ratio = ________

(2) D_1 (or) D_2 = ________ mm

(3) μ = ______

(4) centre distance (C.D) = ______ mm.

(5) ρ = ______ kg/m³.

(6) $(\sigma_{per})_{tensile}$ = ______ MPa.

(7) t (thickness) = ______ mm (or)

E (Young's modulus) ______ GPa (or)

$(\sigma_b)_{per}$ = ______ MPa.

(8) Power = ______ kW.

Some times additional data is also given

such as

(9) (a) S_1 = ______ %

(b) S_2 = ______ %

(10) overload

(or) service factor = K_a = ______

(11) Friction loss at each shaft = ______

Steps for solving conventional questions

(1) Velocity Ratio (V.R) = $\frac{N_2}{N_1}$ = ______

(2) t :- For safe design of belt

$$(\sigma_{max})_{ind.} \leq (\sigma_{per})_{\substack{\text{bending}\\ \text{stress}}}$$

$$\frac{Et}{D_1} \leq (\sigma_{per})_{b.s.}$$

$$\boxed{t \leq \_\_\_\_ \text{ mm}}$$

Eg:- If $t \leq 5.899$ mm [t_{max}].
$t = 5.5$ mm

(3) D_2 :-

$$V.R = \left(\frac{D_1 + t}{D_2 + t}\right)\left(1 - \frac{S}{100}\right)$$

$$\boxed{D_2 = \_\_\_\_ \text{ mm}}$$

Thickness can be taken or can be neglected. value of t is very small thus the value is almost same

(4) Belt velocity (V)

In the absence of slip,

$$V_1 = V = V_2 \quad \text{(convert in m)}$$

$$V = \frac{\pi D_1 N_1}{60} \text{ (or) } \frac{\pi D_2 N_2}{60} \text{ m/sec}$$

(D_1 → mm, D_2 → mm)

$$= \_\_\_\_ \text{ m/sec}$$

In presence of slip,

$$V = V_1\left[1 - \frac{S_1}{100}\right] \text{ (or) } \frac{V_2}{\left(1 - \frac{S_2}{100}\right)}$$

(or) $$V_2 = V\left[1 - \frac{S_2}{100}\right] = \_\_\_\_ \text{ m/sec}$$

(5) m calculation

$$m = \left(\rho \text{ in } \frac{kg}{m^3}\right)\left(\frac{b}{1000}\right)\left(\frac{t}{1000}\right) = \_\_\_\_ b \text{ in kg/m}$$

(6) $T_c = mv^2 = $ ______ b in N.

(7) $T_{max} = \sigma_{per} \times b \times t = $ ______ b in N.

(8) $T_1 = T_{max} - T_c$

$= $ ______ b in N.

(9) $\frac{T_1}{T_2} = e^{(\mu\theta)_{min}}$

$(\mu\theta)_{min} = $ Min. of $(\mu_1\theta_1$ & $\mu_2\theta_2)$

<u>In O.B.D,</u>

$$\theta_1 = \pi - 2\left[\sin^{-1}\left(\frac{D_2 - D_1}{2c}\right) \times \frac{\pi}{180}\right]$$

$$\theta_2 = 2\pi - \theta_1 = \text{\_\_\_\_\_\_}$$

<u>In C.B.D</u>,

$$\theta_1 = \theta_2 = \pi + 2\left[\sin^{-1}\left(\frac{D_2 + D_1}{2c}\right) \times \frac{\pi}{180}\right]$$

$= $ ______

$T_2 = \frac{T_1}{e^{(\mu\theta)_{min}}} = $ ______ b in N.

(10) P.T.C of a belt drive $= (T_1 - T_2)V$

$= $ ______ b in watts.

(11) <u>b :-</u>

(P.T.C. of a belt drive) $\geq P_{Design}$.

$$P_{Design} = \underset{\substack{\downarrow \\ \text{Power to} \\ \text{be transmitted}}}{P_T} \times \underset{\substack{\downarrow \\ \text{overload} \\ \text{factor}}}{K_a} \times \underset{\substack{\searrow \\ \text{frictionless} \\ \text{factor}}}{K_F}$$

$= \_\_\_\_\_\_\_\_ kW$

* Milling m/c shaft → driven

$b \geq \_\_\_\_\_\_ mm$

$b = \_\_\_\_\_\_ mm$

(12) $$L_{O.B.D} = 2c + \frac{\pi}{2}(D_1 + D_2) + \frac{(D_2 - D_1)^2}{4c}$$

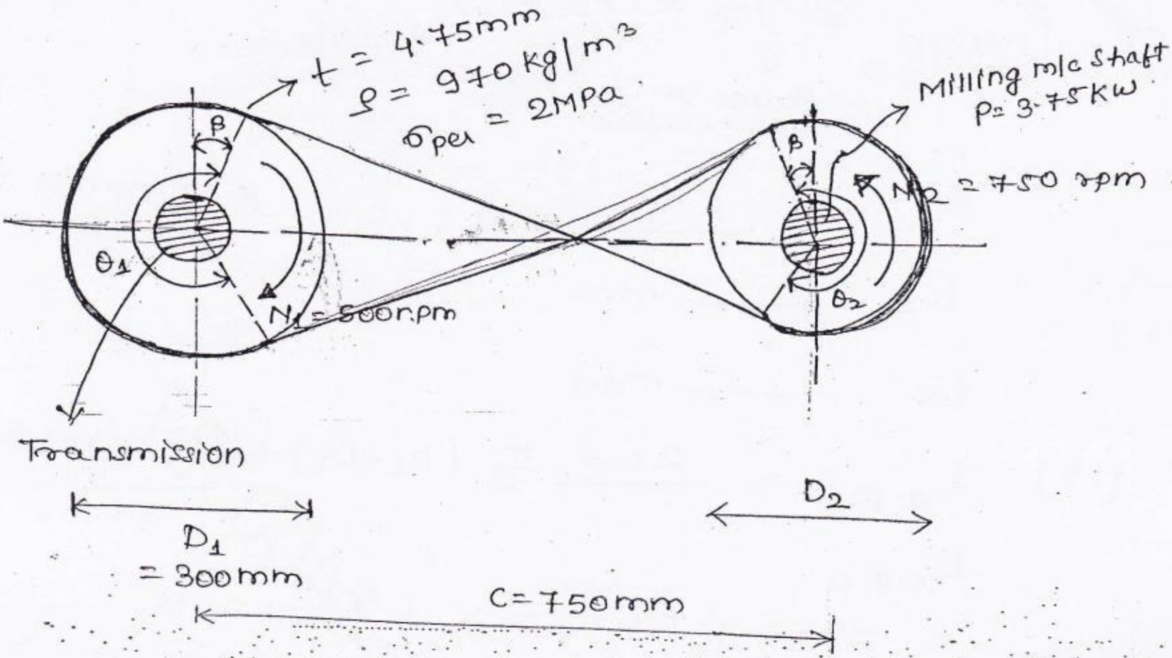
$$L_{C.B.D} = 2c + \frac{\pi}{2}(D_1 + D_2) + \frac{(D_2 + D_1)^2}{4c}$$

Let $L_{O.B.D} = 4992.3$ mm (or) 4.9 m [round off to lower value to get T_{max}]

ES 2002

(Q) A transmission shaft rotating at 500 r.p.m drives a milling m/c which requires 3.75 kW of power at 750 r.p.m. A 300 mm dia cast iron pulley is mounted on the transmission shaft. Initial design proposes a belt of 4.75 mm thickness which has a density of 970 kg/m³. The allowable stress is 2 MPa, two pulleys should rotate in opp. dirn and centre distance of the shaft is 750 mm. $\mu = 0.3$ for both the pulleys. Determine width of the belt.

Sol:- cross belt drive should be used to transmit power from transmission shaft to milling m/c shaft because both the shaft ~~pulleys~~ should rotate in opp. dirn.



(i) $\underline{D_2}$:-

$$V.R = \frac{N_2}{N_1} = \left(\frac{D_1 + t}{D_2 + t}\right)\left(1 - \frac{s}{100}\right)$$

By neglecting the effect of thickness,

$$V.R = \frac{750}{500} = \frac{300}{D_2}(1 - 0)$$

$$\Rightarrow \boxed{D_2 = 200\,mm}$$

(ii) $\underline{V}$:-

In the absence of slip

$$V = V_1 \ (or) \ V_2 = \frac{\pi D_1 N_1}{60} \ (or) \ \frac{\pi D_2 N_2}{60}$$

$$V = \frac{\pi (0.3)(500)}{60} = 7.854 \ m/sec.$$

(iii) $m = \rho \times \frac{b}{1000} \times \left(\frac{t}{1000}\right) = 970 \times \frac{b}{1000} \times \frac{4.75}{1000}$

$= 4.6075 \times 10^{-3}\ b$ in kg/m.

(iv) $T_c = mv^2$

$= 0.285\ b$ in N.

(v) $T_{max} = \sigma_{per} \times b \times t$

$= 2 \times b \times 4.75$

$= 9.5\ b$ in N.

(vi) $T_1 = T_{max} - T_c = \underline{9.216}\ b$ in N.

(vii) $\frac{T_1}{T_2} = e^{(\mu\theta_1 \text{ (or) } \mu\theta_2)}$

$\theta_1 = \theta_2 = \pi + 2\left[\sin^{-1}\left(\frac{300+200}{2\times750}\right) \times \frac{\pi}{180}\right]$

$= 3.821$ rad

$\frac{T_1}{T_2} = e^{(0.3 \times 3.821)} = 3.146$

$T_2 = \frac{T_1}{3.146} = \underline{2.929}\ b$ in N.

(viii) $P.T.C = (T_1 - T_2)V = \underline{49.369}\ b$ in watts.

(ix) $\underline{\underline{b:-}}$

$(P.T.C) \geq P_{Design}$

$(P.T.C) \geq P_T \times \underset{1}{k_a} \times \underset{1}{k_f}$

$$49.369b \geq 3.75 \times 10^3$$

$$\Rightarrow b \geq 75.99 \text{ mm.}$$

$$\boxed{b = 76 \text{ mm}}$$

(Q) Repeat the above problem when both the pulleys are rotating in the same dirn and assumed slip at each pulley is 2%. overload = 25% ($K_a = 1.25$) and friction loss at each shaft = 5%. ($K_f = 1.05$)

[If input power is given then friction loss of both shaft is taken]

22/02/14

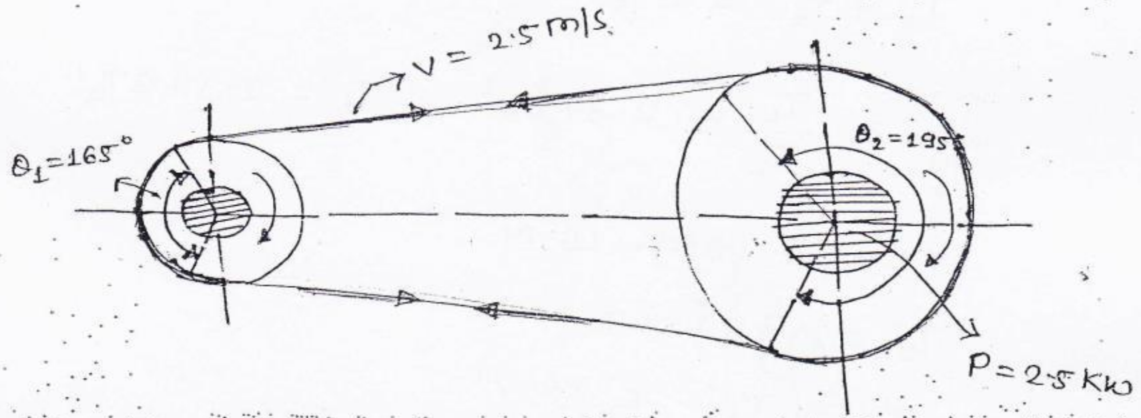
(5) 2.5 kW of power is transmitted by open belt drive. The linear velocity of the belt is 2.5 m/s. The angle of lap on the smaller pulley is 165° and $\mu = 0.3$. It is desired to increase the power to be transmitted. State which of the following methods would be more effective :-

(i) Initial tension (T_0) is increased by 4%.

(ii) Angle of lap is increased by 8% by the use of an idler pulley for the same speed and tension on the tight side.

(iii) coeff. of friction is increased by 8% by suitable dressing to the friction surface of the belt.

Sol:-



* Effect of 'T_c' is neglected because velocity of the belt is less.

* $P = (T_1 - T_2) V$

$$2500 = (T_1 - T_2) \times 2.5$$

$$T_1 - T_2 = 1000\,N \quad \text{—— (I)}$$

$$\frac{T_1}{T_2} = e^{(\mu\theta)_{min}}$$

$$\frac{T_1}{T_2} = e^{0.3 \times 165 \times \pi/180} = 2.372 \quad \text{—— (II)}$$

$T_1 = 1728.25\ N$

$T_2 = 728.25\ N$

$$T_0 = \frac{T_1 + T_2}{2} = \frac{1728.25 + 728.25}{2}$$

$$= \cancel{864.46\ N} = 1228.25\ N.$$

I METHOD

$$T_0' = \cancel{1.08\,T_0 = 1.08 \times 1228.25}$$

$$= 1327.17\ N$$

$$T_0' = \frac{T_1' + T_2'}{2}$$

$$T_1' + T_2' = 2T_0' = 2654.34\ N$$

$$\frac{T_1'}{T_2'} = \frac{T_1}{T_2} = 2.372 \quad , \quad T_1' = 2.372\,T_2'$$

$$T_1' = \cancel{7}\ 1867.16\ N$$

$$T_2' = 787.17\ N$$

$$P' = (T_1' - T_2') \times V'$$

$$P' = (1867.16 - 787.17) \times 2.75$$

$$= 2.699\ KW = 2.7\ KW.$$

% increase in P.T.C

$$= \frac{P' - P}{P} \times 100 = 8\%.$$

II Method

$$\theta' = 1.08\theta$$

$$\theta' = 1.08 \times 165 \times \frac{\pi}{180}$$

$$= 3.11 \text{ radian}.$$

$$\frac{T_1'}{T_2'} = e^{\mu\theta'} = e^{0.3\times 3.11} = 2.542 \quad \text{—— (I)}$$

$$T_1' = T_1 = 1728.25\ N.$$

$$T_2' = \frac{1728.25}{2.542} = 679.878\ N.$$

$$P' = (T_1' - T_2')V$$

$$= (1728.25 - 679.88) \times 2.5$$

$$= 2.62\ KW.$$

$$\%\ \text{Increase in P.T.C} = \frac{P'-P}{P} \times 100$$

$$= 4.788\ \%.$$

III. METHOD

$$\mu' = 1.08\mu = 1.08 \times 0.3 = 0.324.$$

$$\frac{T_1'}{T_2'} = e^{\mu'\theta} = 2.542.$$

$$T_0' = T_0 = \frac{T_1' + T_2'}{2} \Rightarrow T_1' + T_2' = 2T_0.$$

$$\Rightarrow T_1' + T_2' = 2 \times 1228.25 = 2457.72\ N.$$

$$T_1' = \underline{1763.84}\ N$$

$$T_2' = 693.879\ N.$$

$P' = (T_1' - T_2') \times V$

$P' = (1763.84 - 693.879)$

$= 2.674 \, kW$

% increase in P.T.C

$= \frac{P' - P}{P} \times 100\%$

$= \frac{2.674 - 2.5}{2.5} \times 100\%$

$= 6.96\%$

out of above three methods, the best method to increase power transmission capacity (P.T.C) is I^{st} method (i.e, increasing T_0)

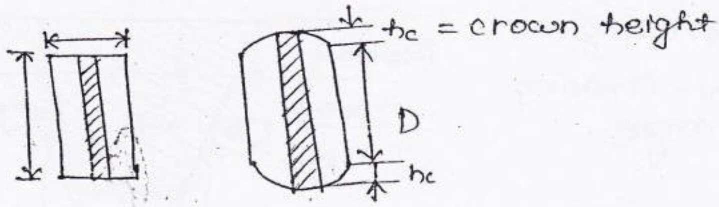
ES'10

(Q) A flat belt drive is required to transmit 10kW of power from a motor running at 1000 rpm. The belt is 15mm thick and a mass density of 0.011 gm/mm³. $(\sigma_t)_{per} = 2.5$ MPa, diam of driver pulley is 250mm whereas speed of the driven pulley is 360 rpm centre distance 1.25 m and $\mu = 0.25$. Find width of the belt for safe working. Ans :- 43.7 mm = 44 mm.

Sol :- By assuming open belt drive.

CROWNING

Method used for reducing slipping of the belt from the pulley surface.



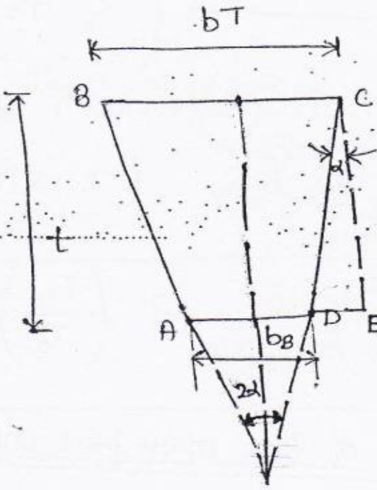
crowning provides a convex surface at the centre of pulley. is known as cr

V- Belt

V- Belts are used to transmit power between two parallel shafts which are rotating in same dirn and; which are located at smaller centre distance

Applications.

Machine tools, automobile radiator fans, Air compressors.



2α = Included angle b/w sides of V-belt
= 38 to 40°

$$A = \frac{1}{2}[b_T + b_B]t$$

Since $\tan\alpha = \frac{DE}{CE}$

$$DE = CE \tan\alpha$$

$$DE = t(\tan\alpha)$$

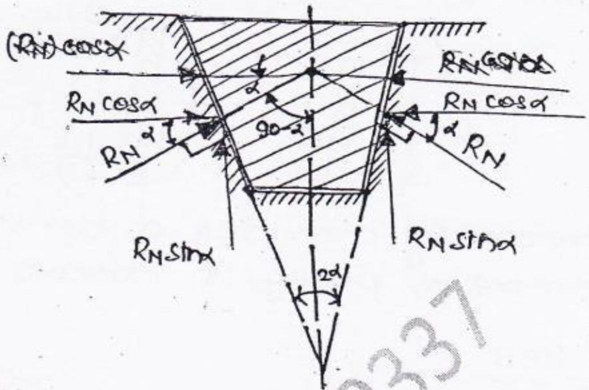
$$\frac{b_T}{2} = \frac{b_B}{2} + DE$$

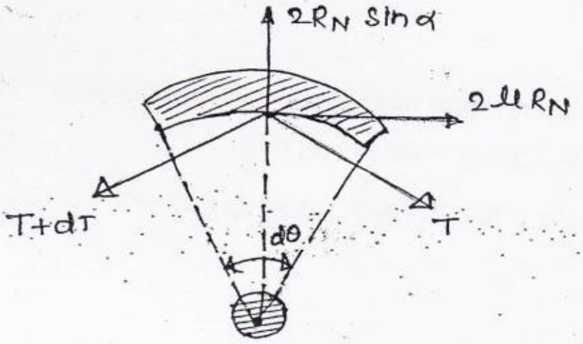
$$\Rightarrow \frac{b_T}{2} = \frac{b_B}{2} + t\tan\alpha$$

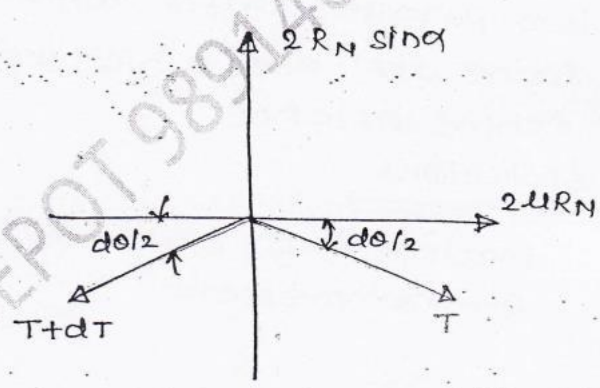
$$\boxed{b_B = b_T - 2t \cdot \tan\alpha}$$

2α = Groove angle

α = Semi-Groove angle.







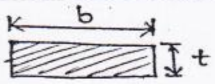
$$\boxed{\frac{T_1}{T_2} = e^{\mu\theta/\sin\alpha}}$$

For a given material, dia. of C.D,

$$\frac{\mu\theta}{\sin\alpha} > \mu\theta \quad (\because \sin\alpha < 1)$$

$$\boxed{\left(\frac{T_1}{T_2}\right)_{V\text{-belt}} > \left(\frac{T_1}{T_2}\right)_{\text{Flat belt}}}$$

Comparision of ~~the~~ open flat belt and V-belt

Parameter	Flat belt	V-belt
(1) CROSS SECTION		
(2) CENTRE DISTANCE [C.D]	Medium C.D.	Smaller C.D.

$\frac{T_1}{T_2}$	$e^{\mu\theta}$	$e^{\mu\theta/\sin\alpha}$
P.T.C	Less	more
Cost	less costlier	more costlier
Slip	More	Less
Idler pulley	Required	Not required
Service life	More	less
Noise	More (Due to existence of a joint)	Quit operation
No. of belts	one belt	Multiple V-belts are required.
Mass per unit length (m)	Less	More

In case of multiple V-belts, even if a single V-belt gets damaged the entire set of the V-belts are to be replaced by a complete set of new V-belts to ensure uniform tensions in all the belts during the power transmission.

V-belt designation

Nominal inside length in mm

TYPE OF V-BELT ⇐

B - 3672 - Gr. 48 [under size belt]

B - 3672 - Gr. 50 [standard size belt] (M.L = N.P.L)

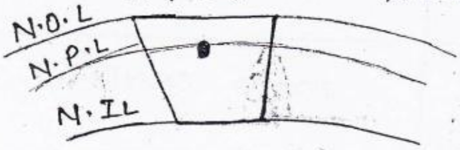
B - 3672 - Gr. 54 [over size belt]

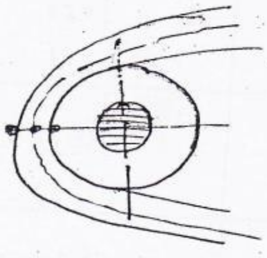
(M.L > N.P.L) Manufacture length is more than the required "

Under size belt $(M.L < N.P.L)$

Manufacture length < Nominal pitch length.

N.OL → Nominal outside length.
N.IL → " inside " .
N.P.L → " pitch " .





Manufactured length (M.L)

$$\boxed{M.L = (N.P.L) \pm [\text{Diff in grade no.s} \times 2.5\,mm]}$$

for over-size (+)
" under size (−)

Each grade no variation gt

$$\boxed{M.L = (N.I.L + k) \pm [\text{Diff in grade No.s} \times 2.5\,mm]}$$

TYPE OF V-BELT	K = N.P.L − N.I.L
A	36
B	43
C	56
D	79
E	92

For 1st Belt

$$M.L = (3672 + 43) - [(50 - 48) \times 2.5]$$

$$= 3715 - 5 = 3710\,mm.$$

II Belt

$$M \cdot L = (3672 + 43) + 0 = 3715 \text{ mm}.$$

III Belt

$$M \cdot L = (3672 + 43) + [(54 - 50) \times 2.5]$$
$$= 3715 + 10 = 3725 \text{ mm}.$$

TYPE OF V-BELT	cross-section		m	COST	Range of Power (kW)
A		I	I	I	0-10
B		N	N	N	5-20
C		C R E	CR E	C R E A	15-30
D		A S E	A S E	S E S	25-40
E		S	S		40-60

* $$L_{NPL} = 2C + \frac{\pi}{2}(D_1 + D_2) + \frac{(D_2 - D_1)^2}{4C} = \underline{\quad\quad}$$

$$NIL = NPL - K.$$

No. of V-belts

$$\text{No. of V-Belts} = \frac{\text{Total power to be transmitted } (P_T) \times K_a}{\text{Power of each belt } (P_{each})}$$

$$= \frac{P_T \times K_a}{(T_1 - T_2)v}$$

$$T_{max} = \sigma_{per} \times A = \underline{\qquad} N.$$

$$m = \rho \times \left(\frac{A^{\to mm}}{10^6}\right) \times 1\,m = \underline{\qquad} kg/m.$$

To convert in m

$$V = V_1 = V_2 = \frac{\pi D_1 N_1}{60} = \underline{\qquad} m/sec$$

$$T_c = mV^2 = \underline{\qquad} N.$$

$$T_1 = T_{max} - T_c = \underline{\qquad} N.$$

$$\frac{T_1}{T_2} = e^{(\mu\theta/\sin\alpha)} \Rightarrow T_2 = \underline{\qquad} N$$

P_{each} (power of each belt)

$$= (T_1 - T_2)V$$

(No. of belt) $n = \dfrac{P_T \times K_a}{P_{each}} = \underline{\qquad}$

No. of V-belts

$$\boxed{\text{No. of V-belts} = \frac{P_T \times K_a}{\left(\begin{array}{c}\text{Rated power}\\ \text{of each V-belt}\end{array}\right) \times K_b \times K_c}}$$

K_b → Arc of contact factor.

K_c → Length correction factor.

Rated power of each V-belt → obtained from manufacturer's catalogue.

* Arc of contact factor is considered in the calculation of no. of V-belts to consider the effect of variation in angle of contact from the assumed angle of contact of 180° in the determination

of rated power.
Length correction factor is considered in the calculation of no. of V-belts to consider the effect of variation in the length of the belt from the standard size of the V-belt.

(Q) It is required to transmit 100 HP at 1440 rpm to a machine running at 350 rpm by no. of V-belts. The diameter of driving wheel is 250 mm. Centre distance 2.25 m. E-section V-belt may be used for which the rated HP transmission capacity is 40.2. Assume service factor, length correction factor, arc of contact factor for the drive arrangement may be taken as 1.02, 1.01, 0.96 resp. The inside length for the belt is 92 mm less than the pitch length. Determine no. of belts required to transmit the power and specify the belt designation.

Sol:-

No. of V-belt

$$= \frac{P_T \times K_a}{(\text{Rated power}) \times K_b \times K_c}$$

$$= \frac{100 \times 1.2}{40.2 \times 0.96 \times 1.01}$$

$$= 3.078$$

$$\boxed{n = 4}$$

$$\frac{N_2}{N_1} = \frac{D_1}{D_2} \Rightarrow \frac{1440}{350} = \frac{250}{D_2}$$

$$\Rightarrow D_2 = \frac{250 \times 350}{1440} = 1028.57 \text{ mm}$$
$$= 1030 \text{ mm}.$$

$L_{N.P.L}$

$$= 2C + \frac{\pi}{2}(D_1 + D_2) + \frac{(D_2 - D_1)^2}{4C}$$

$$= 2 \times 2.5 \times 10^3 + \frac{\pi}{2}(250 + 1030) + \frac{(1030 - 250)^2}{4 \times 2.5 \times 1000}$$

$= \cancel{4571.45 \text{ mm} = 4572 \text{ mm}}$

$= 7071.5 \text{ mm} = 7072 \text{ mm}$

$N.I.L = N.P.L - K = (4572 - 92) \text{ mm} = 6948.0 \text{ mm}$

$$\boxed{'E' = 6948.0 - Gr.50}$$

(8) The following data is given for a V-belt drive connecting 20 kW motor to a compressor. The diameter and speeds are given in the following table

Parameter	Motor Pulley	Compressor pulley
pitch dia (mm)	300	900
Speed (RPM)	1440	880
μ	0.2	0.2

$C.D = 1\text{m}$, $b_T = 22\text{mm}$, $t = 14\text{mm}$, $2\alpha = 40°$,

$\rho = 0.97$ gm/cc.

$T_{allowable} = 850 \text{ N} = T_{max}$.

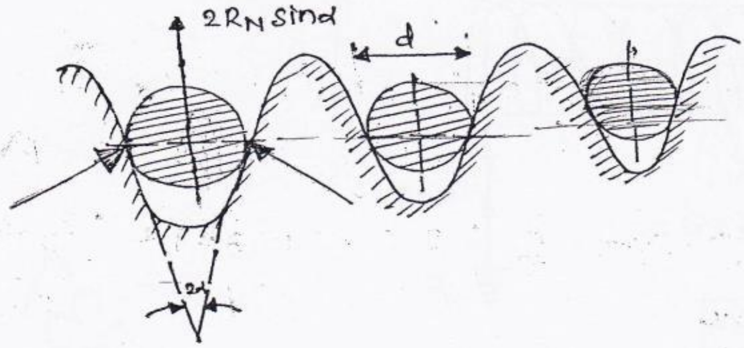
Det. no. of V-belts ?

Ans:-

$b_B = 11.81 \text{mm}$

$n = 1.56 \simeq 2$

Fibre Ropes



d = Dia. of rope.

$$\boxed{\frac{T_1}{T_2} = e^{\mu\theta/\sin\alpha}}$$

(1) $T_{max} = \sigma_{per} \times \frac{\pi}{4} d^2 = \text{\_\_\_\_\_\_} N$

(2) $m = \underset{kg/m^3}{\rho} \times \left(\frac{\frac{\pi}{4} d^2}{10^6} \right) \times 1m = \text{\_\_\_\_\_\_} kg/m.$

(3) $v = v_1 = v_2 = \frac{\pi D_1 N_1}{60}$ (or) $\frac{\pi D_2 N_2}{60} = \text{\_\_\_\_} m/sec$

(4) $T_c = m v^2 = \text{\_\_\_\_\_\_} N.$

(5) $T_1 = T_{max} - T_c = \text{\_\_\_\_\_\_} N$

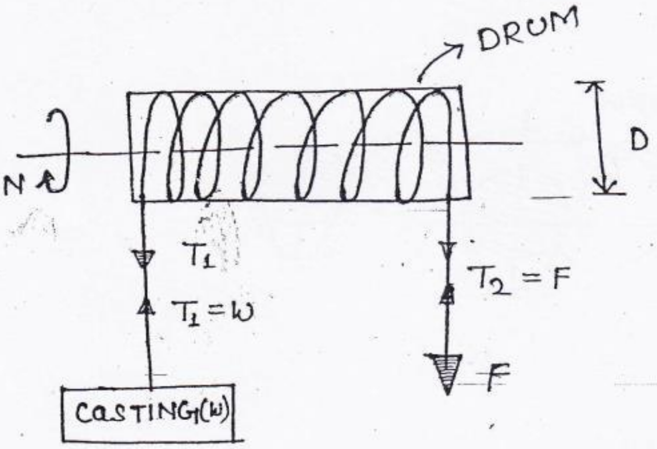
(6) $\frac{T_1}{T_2} = e^{(\mu\theta/\sin\alpha)} \Rightarrow T_2 = \text{\_\_\_\_\_} N.$

(7) Power of each rope

$P_{each} = (T_1 - T_2) v = \text{\_\_\_\_\_} watts.$

(8) $$\boxed{n = \frac{P_T \times K_a}{P_{each}}}$$

2α = Groove angle = sheave angle.



$$\frac{T_1}{T_2} = e^{\mu\theta \to 2\pi n \to \text{no. of turns}}$$

$\Rightarrow T_2 =$ ____

$\Rightarrow T_2 = F =$ ____

$P = (T_1 - T_2)V =$ ____

ES-11

(Q) A rope pulley is designed to transmit 30 kW, dia. of pulley = 360 mm, speed = 120 rpm, Angle of groove = 45°, Angle of Lap on smaller pulley = 170°, $\mu = 0.27$, no. of ropes = 10, mass of the rope = $55c^2$ kg/m,

Max and working tension (T_{max}) is limited to $125c^2$ KN where c is circumference of rope in metres. Find
(i) Initial tension.
(ii) Dia. of each rope.

Sol:→ $P_T = 30$ kW, D = 360 mm, N = 120 rpm,
$2\alpha = 45°$, $\theta_1 = 170°$, $\mu = 0.27$,
n = 10, $m = 55c^2$ kg/m, $T_{max} = 125c^2$ KN.

$$V_1 = \frac{\pi D N}{60} = \frac{\pi \times 360 \times \cancel{120}^{2}}{\cancel{60}} = 720\pi$$

$= 2261.946$ mm/s.

$$\theta_1 = 170 \times \frac{\pi}{180} = 2.967 \text{ radians}$$

$c = \pi d$ in m
↓
dia. of rope.

(a) $T_0 = ?$ (b) $d = ?$

(1) $$n = \frac{P_T \times K_a}{P_{each}} \Rightarrow 10 = \frac{30 \times 1}{P_{each}}$$

$$\Rightarrow \boxed{P_{each} = 3\ KW}$$

(2) $$P_{each} = (T_1 - T_2) v = (T_1 - T_2)\, 2.261.$$

$$\Rightarrow 3000 = (T_1 - T_2)\, 2.261$$

$$\Rightarrow \boxed{T_1 - T_2 = 1326.29\ N} \quad \text{—(I)}$$

(3) $$\frac{T_1}{T_2} = e^{\mu\theta / \sin\alpha} \Rightarrow \frac{T_1}{T_2} = e^{\frac{0.27 \times 170 \times \pi}{180 \times \sin 22.5}}$$

$$\boxed{\frac{T_1}{T_2} = 8.11} \quad \text{(II)}$$

Solving (I) & (II), we get

(4) $T_1 = 1512.78$ N.

$T_2 = 186.48$ N.

(5) $$T_c = mv^2 = 55\, c^2 \times (2.261)^2$$
$$= 281.402\, c^2\ \text{Kgm/s}^2$$
$$= 281.402\, c^2\ N.$$

(6) $T_{max} = T_1 + T_c$

$\Rightarrow 125c^2 \times 1000 = 1512.78 + 281.402c^2$

$\Rightarrow c = 0.1101 \text{ m}$.

$c = \pi d = 0.1101 \text{ m}$

$\Rightarrow d = 35.04 \text{ mm} \simeq 36 \text{ mm}$.

$c = \pi(0.036)\text{m}$

$= 0.11309 \text{ m}$.

$T_c = 281.402\, c^2$

$= 281.402 \times (0.11309)^2$

$= 3.599 \text{ N}$.

$$T_0 = \frac{T_1 + T_2 + 2T_c}{2}$$

$$= \frac{1512.78 + 186.48 + 2(3.599)}{2}$$

$= 853.23 \text{ N}$.

(8) The grooves on the pulleys of a multiple rope drive has an angle of 50° and accomodates ropes of 22 mm dia. having a mass of 0.8 kg/m. safe operating tension = 1200N, two pulleys are of equal ~~tensio~~ size (means $\theta = 180°$). The drive is designed for maxm power conditions, speed of both the pulleys is 180 rpm. Determine dia. of the pulleys and no. of ropes when power to be transmitted is 150 kW and $\mu = 0.25$.

<u>sol</u> → $T_{max} = 1200\text{N}$, $2\alpha = 50°$,

$d = 22\text{mm}$, $m = 0.8 \text{ kg/m}$

$\theta = 180^\circ$, $N = 180$ rpm, $P_T = 150$ kW, $\mu = 0.25$

D $d = ?$, $n = ?$

$\cancel{N \neq \frac{\pi DN}{60}}$ (1) $v_{max} = \sqrt{\frac{T_{max}}{3m}}$

$\cancel{P_{max} =}$ $= \sqrt{\frac{1200}{3 \times 0.8}} = 22.36$ m/sec.

(2) $T_c = \frac{T_{max}}{3} = \frac{1200}{3} = 400$ N.

(3) $T_1 = 2T_c = 2 \times 400 = 800$ N.

(4) $\frac{T_1}{T_2} = e^{\mu\theta/\sin\alpha}$

$\Rightarrow T_2 = \frac{T_1}{e^{(0.25 \times \pi/\sin 25^\circ)}} = 124.73$ N.

(5) $(P_{max})_{\text{each rope}} = (T_1 - T_2)\, v_{max}$

$= (800 - 124.73) \times 22.36$

$= 15.09 \times 10^3$ watts $= 15.09$ kw.

(6) $n = \frac{P_T \times K_a}{(P_{max})_{each}} = \frac{150 \times 1}{15.09}$

$= 9.94$

$\simeq 10$ ropes.

(7) $v_{max} = 22.36 = \frac{\pi D \times \cancel{180}^{30}}{\cancel{60}}$

$\Rightarrow D = \frac{22.36}{30\pi} = 2.3724\ m$.

$\Rightarrow \boxed{D = 2.38\ m}$

[Gear (F.B.D)
T.O.F
S.F.D, B.M.D]

DESIGN OF SHAFTS

- Horizontal belt drive → centres of shaft lie in horizontal plane.
- vertical belt drive → centres of shaft lie in vertical plane.

Spindle is always hollow but shaft can be hollow or solid.

Design of shafts

(1) Torque is transmitted by the shaft :-

$$T_m = \frac{P \times 60}{2\pi N} \times 10^6 = \_\_\_\_\_\_ \text{N-mm}$$

(P → kW)

$$T_{Design} = T_m \times K_a = \_\_\_\_\_\_ \text{N-mm}$$

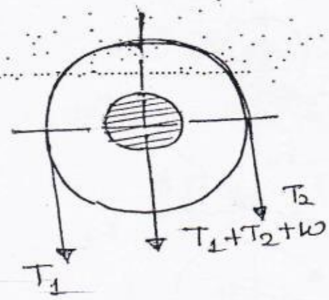
(2) Draw the torque diagram.

(3) Draw the free body diagram (F.B.D)

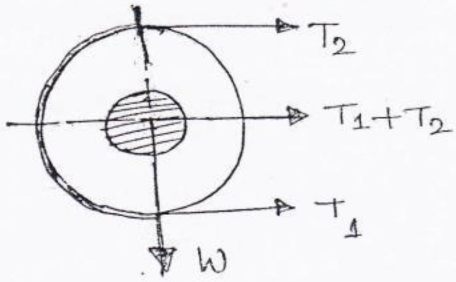
(a) vertical Belt Drive (V.B.D)

Driven pulley rotation depends upon motor

Driven pulley rotation depends upon tension.



(b) Horizontal belt Drive



$$T_{Design} = (T_1 - T_2)R$$

$$T_1 - T_2 = \_\_\_\_\_\_\_\_ N$$

$$\frac{T_1}{T_2} = e^{\mu\theta \to \pi \text{ (radian)}}$$ (If $\frac{T_1}{T_2}$ is not given then use this formula)

$$T_1 = \_\_\_\_ N$$

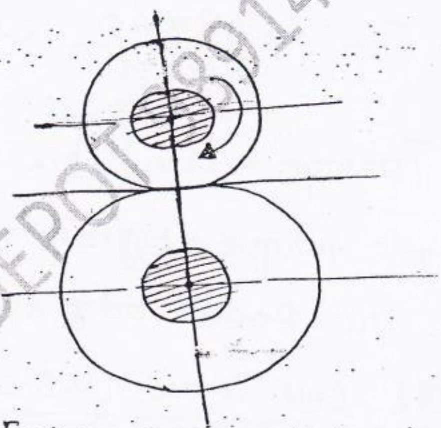
$$T_2 = \_\_\_\_ N$$

(a) **Vertical Gear Drive (F.B.D)**

$F_R \to$ In the dirn of power transmission.

$\phi \to$ Pressure angle

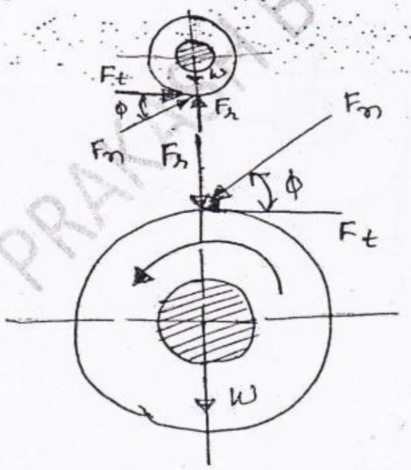
$F_n \to$ force by which pinion is driving gear.



Here, dirn of F_R is downward [Always towards line of centre]

$F_t \to$ Dirn in the dirn of rotation.

Resultant of F_R and F_t is F_n.

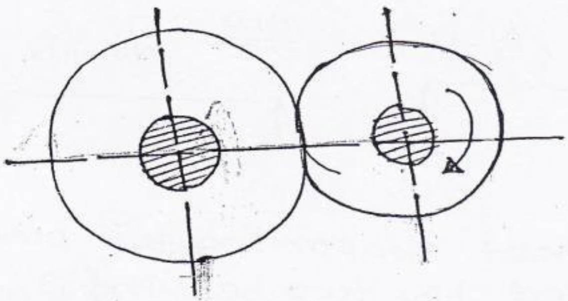


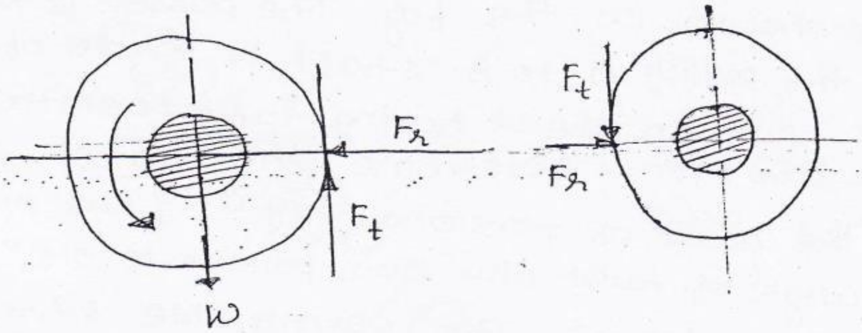
Rkns on pinion is equal and opp.

Here, line of centre lies in vertical plane thus dirn of F_R lies in same plane.

(b) Horizontal Gear Drive (H.G.D)

F_t is responsible for torque transmission.





$$F_t = \frac{2T_D}{D} = \_\_\_\_\_\_ N.$$

$$F_r = F_t \tan\phi = \_\_\_\_\_\_ N$$

(4) Draw the horizontal loading diagram (H.L.D)

Shafts are supported by bearing.
Long bearing → fixed support
Short bearing act as simple support.

(5) Draw the horizontal bending moment diagram (HBMD).

(6) Draw the vertical loading diagram (VLD)

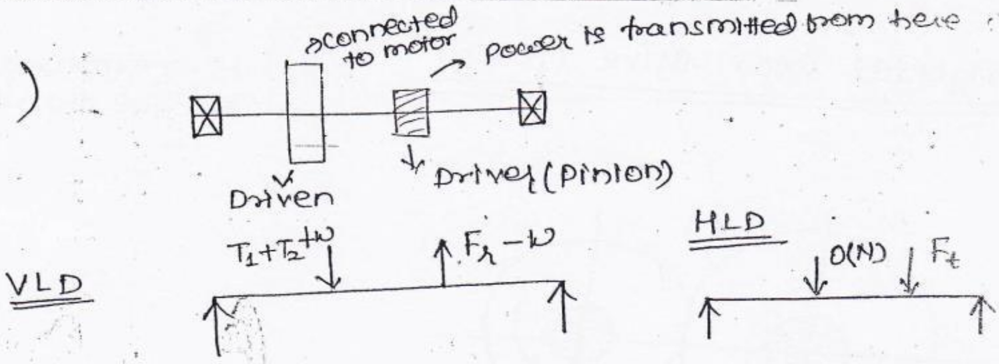
(7) Draw the vertical bending moment diagram (VBMD)

(8) Draw the Resultant " " " (RBMD).

(9) $T_e = \sqrt{(K_b M_R)^2 + (K_t T_D)^2} = \frac{\pi}{16} d^3 \tau_{per}$.

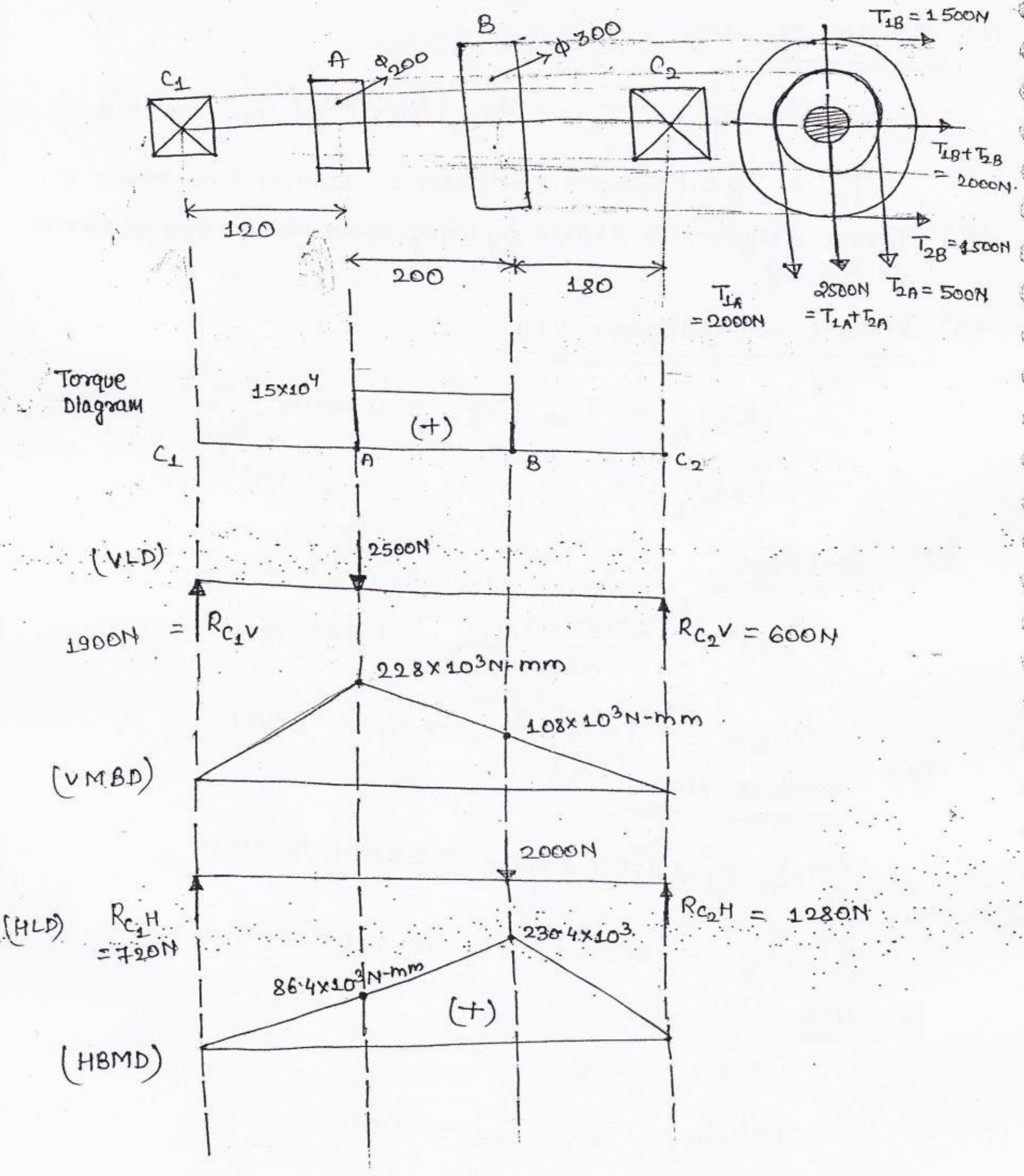
$$d = \_\_\_\_\_\_ mm.$$

(9)



ES 1996

(9) A transmission shaft supporting two pulleys A and B are mounted b/w two bearing C_1 and C_2 as shown in the fig. The power is transmitted from the pulley A to B. shaft is made of plane ~~cross~~ carbon steel having $S_{ut} = 600$ MPa, $S_{yt} = 380$ MPa. Determine the dia. of shaft on the basis of torsional rigidity, if permissible angle of twist b/w two pulleys is $0.5°$ and $G = 80$ KN/mm^2. The permissible shear stress $\tau_{per} = 30\%$ of S_{yt} (or) 18% of S_{ut} whichever is min. check for safety


B
A
φ200
φ300
C1
C2
T1B = 1500N
T1B+T2B = 2000N.
T2B = 1500N
120
200
180
T1A = 2000N
2500N = T1A+T2A
T2A = 500N
Torque Diagram
15x10^4
(+)
C1
A
B
C2
(VLD)
2500N
1900N = RC1V
RC2V = 600N
228x10^3 N-mm
108x10^3 N-mm
(VMBD)
2000N
(HLD)
RC1H = 720N
RC2H = 1280N
230.4x10^3
86.4x10^3 N-mm
(+)
(HBMD)

(1) T_A and T_B :-

$$T_A = T_B = (T_{1A} - T_{2A}) R_A \text{ (or) } (T_{1B} - T_{2B}) R_B$$

$$T_A = T_B = (2000 - 500) 100 = 15 \times 10^4 \text{ N-mm}.$$

(2) Torque diagram is drawn by using above values and as shown in the fig.

(3) Vertical loading diagram (VLD)

$$(W_V)_A = T_{1A} + T_{2A} = 2500 \text{N}.$$

$$(W_V)_B = 0.$$

(4) Reactions

$$R_{C_1V} = \frac{2500 \times 380}{500} = 1900 \text{ N}.$$

$$R_{C_2V} = (2500 - 1900) \text{N} = 600 \text{N}.$$

(5) Vertical BMD

$$(M_V)_A = 1900 \times 120 = 228000 \text{ N-mm}$$

$$(M_V)_B = 600 \times 180 = 108 \times 10^3 \text{ N-mm}$$

(6) HLD

$$(W_H)_A = 0$$

$$(W_H)_B = T_{1B} + T_{2B} = 2000 \text{N}.$$

(7) $$R_{C_1H} = \frac{2000 \times 180}{500} = 720 \text{N}.$$

$$R_{C_2H} = 1280 \text{ N}.$$

(8) HBMD

$$(M_H)_A = 720 \times 120 = 86.4 \times 10^3 \text{ N-mm}.$$

$$(M_H)_B = 1280 \times 180 = 230.4 \times 10^3 \text{ N-mm}.$$

(9) RBMD

$$(M_R)_A = \sqrt{(M_V)_A^2 + (M_H)_A^2}$$

$$= 243.8 \times 10^3 \text{ N-mm}.$$

$$(M_R)_B = \sqrt{(M_V)_B^2 + (M_H)_B^2}$$

$$= 254.45 \times 10^3 \text{ N-mm}.$$

(10) ~~Critical cross-section of the shaft~~

critical cross-section is the c/s where pulley B is mounted where resultant B.M is maxm and torque is also maxm. Hence, equivalent ~~torque is to be calculated at B.~~

$$(T_e)_B = \sqrt{[k_b (M_R)_B]^2 + [k_t T_B]^2}$$

Equivalent torque is calculated by using M.S.S.T.

$k_b = 1$; $k_t = 1$.

(11) Dia. of the shaft by using torsional rigidity criterion

$$\theta_{max} \le \theta_{per}$$

$$\frac{TL}{GJ} \le 0.5 \times \frac{\pi}{180}$$

$$\frac{15 \times 10^4 \times 200}{80 \times 10^3 \times \frac{\pi}{32} d^4} \le 0.5 \frac{\pi}{180}$$

[Pure torsion is present only in the portion AB thus L is taken only for AB].

$$\Rightarrow d^4 \ge 25.72 \text{ mm} \Rightarrow d = 30 \text{ mm}.$$

(12) check for safety w.r.t strength criterion

$$\tau_{per} = \text{Min}\left[(0.3 \times S_{yt}) \text{ \& } (0.18 \times S_{ut}) \right]$$

$$= 108 \text{ MPa}.$$

(13) $$(\tau_{max})_{ind} = \frac{16 (T_e)_B}{\pi d^3} = 55.7 \text{ MPa}.$$

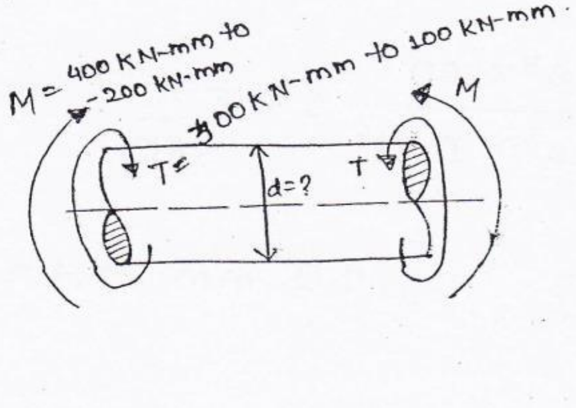
conclusion

Max^m shear stress induced in the shaft is less than τ_{per}. Hence, the dia. obtained by torsional rigidity criterion is also safe w.r.t. strength criterion.

Design of shaft under fluctuating loads

(8) A hot rolled steel shaft is subjected to a torsional load that varies from 300 kN-mm (CW) to 100 kN-mm (ACW). Applied BM at a critical section varies from 400 kN-mm to −200 kN-mm, the shaft is of uniform dia. (means prismatic) and no keyway is present at the critical section. Determine required shaft dia. by taking a FOS of 1.5 for the material S_{ut} = 560 MPa and S_{yt} = 420 MPa. (endurance limit) Design stress = 280 MPa. Assume modification factor as 0.62, size correction factor 0.85 (K_a), load factor for bending (K_c) is 1 and load factor for for torsion is 0.577.

Sol :-



$k_f = k_t = 1$ ($\because$ no discontinuity)

$N = 1.5$

$S_{ut} = \sigma_{ut} = 560 MPa$.

$S_{yt} = \sigma_{Yt} = 420 MPa$.

Design stress $= \sigma_e^* = 280 MPa$.

Modification factor $= k_b = 0.62$.

size correction factor $= k_a = 0.85$.

load factor for bending $= k_c = 1$.

load " " torsion $= 0.577$.

When Bending moment acting alone

$$M_{max} = 400 \ KN-mm.$$

$$M_{min} = -200 \ KN-mm.$$

$$M_{mean} = \frac{400-200}{2} = 100 \ KN-mm.$$

$$\frac{M_{max}-M_{min}}{2} = M_v = \frac{400+200}{2} = 300 \ KN-mm$$

$$\sigma_m = \frac{M_m}{Z_{N.A}} = \frac{32 M_m}{\pi d^3}$$

$$= \frac{32 \times 100 \times 10^3}{\pi d^3} = \frac{1.018 \times 10^6}{d^3} MPa.$$

$$\sigma_v = \frac{32 M_v}{\pi d^3} = \frac{32 \times 300 \times 10^3}{d^3} MPa$$

$$= \frac{3.055 \times 10^6}{d^3} MPa.$$

By using soderberg eq^n

$$(\sigma_{eq}) = \sigma_m + \frac{K_f \sigma_v \sigma_{yt}}{\sigma_e}$$

$$\sigma_e = \sigma_e^* \, K_a \, K_b \, K_c$$

$$= 280 \times 0.85 \times 0.62 \times 1$$

$$= 147.56 \, MPa$$

$$(\sigma_{eq}) = \frac{1.018 \times 10^6}{d^3} + \frac{1 \times 3.055 \times 10^6 \times 420}{d^3 \times 147.56}$$

$$\sigma_{eq} = \frac{9.71 \times 10^6}{d^3} \, MPa \quad \text{—— (I)}$$

when T.M is acting alone

$$T_{max} = 300 \, KN\text{-}mm$$

$$T_{min} = -100 \, KN\text{-}mm$$

$$T_m = \frac{T_{max} + T_{min}}{2} = 100 \, KN\text{-}mm$$

$$T_v = \frac{T_{max} - T_{min}}{2} = 200 \, KN\text{-}mm$$

$$\tau_m = \frac{16 T_m}{\pi d^3} = \frac{16 \times 100 \times 10^3}{\pi d^3}$$

$$\tau_m = \frac{0.509 \times 10^6}{d^3} \, MPa$$

$$\tau_v = \frac{16 T_v}{\pi d^3} = \frac{16 \times 200 \times 10^3}{\pi d^3}$$

$$\tau_v = \frac{1.018 \times 10^6}{d^3} \, MPa.$$

$$\tau_e^* = \sigma_e \, K_a \, K_b \, K_c$$

$= 280 \times 0.85 \times 0.62 \times 0.577$

$= 85.58$ MPa.

By using soderberg's eqn,

$$\tau_{eq} = \tau_m + \frac{k_f \, \tau_v \, \tau_{ys}}{\tau_e}$$

$$\tau_{ys} = S_{ys} = \frac{S_{yt}}{2} = 210 \text{ MPa}.$$

$$\tau_{eq} = \frac{3 \times 10^6}{d^3} \text{ MPa} \quad \text{--- II}$$

Design of shaft (d)

Shaft dia. is obtained by using MSST because it is made up of ductile material and subjected to combined stresses.

$$\tau_{per} = \frac{S_{ys}}{N} = \frac{S_{yt}}{2N} = \frac{1}{2}\sqrt{(\sigma_{eq})^2 + 4(\tau_{eq})^2}$$

$$\Rightarrow \frac{420}{1.5} = \sqrt{\left(\frac{9.71 \times 10^6}{d^3}\right)^2 + 4\left(\frac{3 \times 10^6}{d^3}\right)^2}$$

$$\Rightarrow d = 34.42 \text{ mm}$$

$$\boxed{d = 35 \text{ mm}}$$

Design procedure used in spur Gears

INPUT DATA :-

(1) $P = x$ kW at Y rpm

(2) $G =$ ______

(3) $\phi =$ ______

(4) $[\sigma_{b1}] =$ ______ MPa ; $[\sigma_{b2}] =$ ______ MPa.

(5) $Y_1 =$ ______ ; $Y_2 =$ ______

(or) $y = 0.124 - \frac{0.512}{z} \Rightarrow \phi = 20°$ (full Depth)

(6) $C_v = \frac{3}{3+v}$ (or) $\frac{3+v}{3} \Rightarrow v \leq 10$ m/sec

(7) $\sigma_{es} =$ ______ MPa (surface endurance limit)

(8) $E =$ ______ MPa ; $E_2 =$ ______ MPa.

Optional data :-

$Z_1 =$ ______

$k_a =$ ______

Step 1 :-

(1) $T_1 =$ torque to be transmitted by the pinion.

$$T_1 = \frac{P \times 60}{2\pi N} \times 10^6 = \text{\_\_\_\_\_\_ N-mm}$$

$N \rightarrow$ rpm.

(2) Design torque,

$[T_D]_1 =$ Design torque for pinion.

$$[T_D]_1 = T_1 \times K_a = \text{\_\_\_\_\_\_ N-mm}$$

(3) $(Z_1)_{min}$ = Min. no. of teeth on the pinion to avoid interference.

$$(Z_1)_{min} = \frac{2a_w}{\sin^2\phi} = \_\_\_\_\_\_$$

$a_w \Rightarrow$ Addendum coefficient.

$$\boxed{a_w \times m = a}$$

$m \rightarrow$ module
$a \rightarrow$ addendum.

For full depth teeth, $(a = m)$

$$\boxed{a_w = 1}$$

For stub tooth,

$$\boxed{a = 0.8m \Rightarrow a_w = 0.8}$$

$\phi = 20$ (Full depth)

$$(Z_1)_{min} = \frac{2 \times 1}{\sin^2 20} = 17.09$$

Z_1 = Actual teeth provided on the pinion

$$\boxed{Z_1 = 18}$$

$\phi = 20°$ (stub)

$$(Z_1)_{min} = \frac{2 \times 0.8}{\sin 20°} = 13.67$$

$$\boxed{Z_1 = 14}$$

(4) $G = \frac{N_1}{N_2} = \frac{Z_2}{Z_1}$

$$Z_2 = GZ_1 = \_\_\_\_\_\_$$

(5) Module calculation (m)

$$m \geq 1.26 \sqrt[3]{\frac{[T_D]_1}{([\sigma_b]Y)_{w.G}\,\psi Z_1}}$$

Module is determined by using the following eqn which is obtained from beam strength eqn of weaker gear.

$$\boxed{m \geq 1.26 \sqrt{\frac{[T_D]_1}{([\sigma_b]Y)_{w.G}\,\psi Z_1}}}$$

where, $w.G \to$ weaker gear

$\psi \to$ constant $= \frac{b}{m}$; $8 \leq \psi \leq 14$

$\psi = 10 \Rightarrow b = 10\,m$

$$([\sigma_b]Y)_{w.G} = \text{min. of} \left\{ [\sigma_{b1}]Y_1 \ \& \ [\sigma_{b2}]Y_2 \right\}$$

$m \geq$ ______ mm

(6) Dimensions of Gear tooth

$D_1 = mZ_1 =$ ______ mm

$D_2 = mZ_2 =$ ______ mm

$b = \psi m =$ ______ mm

when Beam strength > Dynamic load [Module is safe]

(7) Beam strength of weaker gear

$$F_s = ([\sigma_b]Y)_{w.G}\, b\, m = \text{______ N.}$$

(8) F_d = Dynamic load

$$F_d = F_t \times C_v \quad (\text{or}) \quad \frac{F_t}{C_v}$$

C_v should be > 1

C_v should be < 1

$$F_t = \frac{2[T_D]_1}{D_1} \quad (\text{or}) \quad \frac{2[T_D]_2}{D_2}$$

$$= \_\_\_\_\_\_ \text{ N.}$$

$$v_1 = v_2 = v = \frac{\pi D_1 N_1}{60} \text{ m/sec}$$

$$C_v = \frac{3+v}{3} \quad (\text{or}) \quad \frac{3}{3+v} = \_\_\_\_\_\_$$

$$F_d = \_\_\_\_\_\_ \text{ N.}$$

- If $F_d \leq F_s \Rightarrow$ Design is safe w.r.t Bending failure.
- If $F_d > F_s \Rightarrow$ Design is unsafe w.r.t bending failure.

(9) New module (m)

$$F_s = F_d$$

$$\left([\sigma_b]Y\right)_{w \cdot G} (\psi m)(m) = F_d$$

$$m = \_\_\_\_\_\_ \text{ mm.}$$

(10) $D_1 = m z_1 = \_\_\_\_\_\_$ mm

$D_2 = m z_2 = \_\_\_\_\_\_$ mm

$b = \psi m = \_\_\_\_\_\_$ mm.

(10) $F_s = \left[(\sigma_b)\cdot Y\right]_{w.G} \; b\, m =$ ______ N

(11) $F_d = F_t \times C_v$

$$F_t = \frac{2[T_{D_1}]}{D_1} = \text{\_\_\_\_\_\_}$$

$$v = \frac{\pi D_1 N_1}{60} = \text{\_\_\_\_\_\_ m/sec}$$

$$C_v = \frac{3+v}{3} = \text{\_\_\_\_\_\_}$$

$F_d =$ ______ N.

$F_d < F_s \Rightarrow$ Design is safe w.r.t bending.

If surface endurance limit is given then use this procedure step.

(12) Wear strength (F_w)

Wear strength is calculated w.r.t pinion because chances of wear failure of pinion is more as its r.p.m is more.

$$F_w = D_1 Q K b$$

$$Q = \frac{2G}{G \pm 1} = \text{\_\_\_\_\_\_}$$

(+) for External gears.

(−) " internal "

K = Material combination factor in N/mm^2.

$$\boxed{K = \frac{(\sigma_{es})^2 \sin\phi \left[\frac{1}{E_1} + \frac{1}{E_2}\right]}{1.4}} = \text{\_\_\_\_ } N/mm^2$$

* $F_w =$ ______ N

$F_d < F_w$ (safe design)

F_w amount of dynamic load it can withstand without any ~~bending~~ wear failure.

$$\boxed{F_d \le F_w}$$

Design w.r.t wear failure.

Design for m eqn

$$F_d \le F_s$$

$$(F_t)_{max} \le ([\sigma_b] Y)_{w.G} \psi . m . m$$

Max^m tangential load

$$\frac{2[T_D]_1}{D_1} \le ([\sigma_b] Y)_{w.G} \psi m^2$$

$$\frac{2[T_D]_1}{m z_1} \le ([\sigma_b] Y)_{w.G} \psi m^2$$

$$\frac{2[T_{D1}]}{([\sigma_b] Y)_{w.G} \psi z_1} \le m^3$$

$$m^3 \ge \frac{2[T_{D1}]}{([\sigma_b] Y)_{w.G} \psi z_1}$$